# AI in mathematics

## On guided intuition and the new environment of calculation

*Gérard Biau in conversation with Anna Echterhölter, January 28, 2023*

**Echterhölter:** As a mathematician by training, as a professor at the Probability, Statistics, and Modeling Laboratory (LPSM), and as the director of the Sorbonne Center for Artificial Intelligence (SCAI), you are in a unique position to observe current changes within various sciences in response to AI. Given that research topics and new instruments arrive and fade in all of the sciences, would you say AI is currently changing mathematics?

**Biau:** There are different ways of answering this question. About 20 years ago, mathematics was a kind of a solo science, that you did alone in a library. Today, we are very strongly influenced by AI tools (of course this does not only concern mathematicians) which are very much having effects on mathematics in the sense that now mathematicians use Google, they communicate with each other via email, we have recommendation systems to find papers on the web, etc. Mathematicians have an aptitude and an openness for using AI tools in their research. That is the first important point.

The second point that must be underlined, is that machine learning is a real game changer for mathematicians because it is an experimental science. In mathematics, when we wrote papers 20 years ago, there were only one or two authors, it was a kind of confidential science.

Today, in machine learning papers, mathematics is part of the paper and there are five, six, seven authors, because it's experimental. The mathematician here is often part of an interdisciplinary team. This is very interesting, because, as a result, mathematicians come to play a new role.

We cannot ignore the impact of machine learning on mathematics. It can reinvent the field with new vocabulary and tools, and the new generation is totally free with this new system.

The third point is that it is an auspicious moment for mathematics. There are new fields of mathematics which have been created because of AI and machine learning. For example, until 20 years ago, high dimension was totally absent from the statistics world, or even from the world at large. Everything changed around the year 2000, when genetic data arrived. With genetic data, you have millions of dimensions that are much larger than the sample size. So new tools were invented, new seminars were created, new papers were written on this and new methods, of course, were invented for high-dimensional phenomena.

To continue, I could also cite optimization. These neural nets we are talking about need optimization to find the right parameters to make the right decision. When you were an optimizer in the 90s, it was kind of hard to find an academic position. But today, if you are an optimizer, you have offers from universities and IT companies. AI is also a game changer from this point of view. It is the same with topological data analysis, which is the field that analyses the geometrical properties of the cloud points (its density, the number of components…). Topological data analysis is something new that did not exist 15 years ago.

This is an entirely new field which is newly created in mathematics and which is an emerging area for the understanding of the properties of big data sets today. I could also talk about the so-called "physics-informed learning", a totally new field that merges data science with scientific computing. It is a mixture of differential equations, evolutionary phenomena and machine learning. These two fields are merging because now we have data, we have algorithms and we have a new point of view on these topics. New areas of mathematics have been created due to AI and this is an important lesson.

Fourthly and finally, something is emerging today that is not so present in the mathematics community, but that is there. I am speaking about the use of AI to prove new theorems or to guide intuition. This is a field that is very, very important and highly interesting. I am not certain if it will transform mathematics, but what I'm certain about, is that it will help mathematicians develop new tools.

**Echterhölter:** For non-mathematicians it can come as a surprise how much talk there is about intuition within this most exact of all disciplines. You have brought up the important category of "guided intuition", which describes quite a fundamental change of a mathematical research practice. Hitherto, finding new proofs has been associated with pen and paper, walks in the woods, sus-

tained periods of concentration in solitary silence and maybe a chalkboard. Then came digital tools and especially AI. What happens to this type of problem solving? Does it give way to new modes, settings and tools of finding proof in mathematics entirely? In particular, you have mentioned applications that assist in mathematics like "Minerva". Could you elaborate on the role that this new tool, or mathematical AI, plays for mathematicians in "guiding their intuition"?

**Biau:** There is this tool, Minerva, which is able to solve very simple problems, say, provide proofs and answer elementary questions (Dyer/Gur-Ari 2022). It can be defined as a language model that is capable of solving mathematical problems and scientific questions using step by step reasoning. There is no tool today that can really solve very complex problem, but Minerva is an interesting step in this direction, and who knows what will happen in the next couple of years.

Beyond Minerva, today we have companies such as DeepMind that are interested in using machine learning for guiding intuition. In this regard, there was an important paper in *Nature* (Davies et al. 2021), in which they used machine learning to propose new relations in pure mathematics, thus allowing the mathematicians to verify relations suggested by the computer. The authors use data to discover potential patterns and relations between mathematical objects and use these observations to guide intuition and propose conjectures. This is a new type of collaboration between AI and mathematics.

In October 2022, just one weekend before the Paris conference, there was another breakthrough, another DeepMind paper in *Nature* (Fawzi et al. 2022). This time, the computer used machine learning to find a new way to multiply matrices. It is crucial to stress that multiplying matrices is a very important concept for machine learning, as it is full of matrices! It is very difficult to have efficient and clever ways to multiply matrices in order to save time and space. Now, DeepMind's algorithm was able to find a new way of multiplying two 4x4 matrices, which is already something, suggesting new algorithms. This is an important step.

You asked me if my discipline changed because of this? The answer is no, at this point. But maybe one day, a computer will probably help mathematicians. Peter Scholze, a German mathematician and a Fields Medalist in 2018, a great man, is working at the interface between algebraic geometry and topology. One of his recent proofs was verified by a computer and also presented in *Nature* in 2021 (Castlevecchi 2021). That was a big achievement, because it was the first

time that the full proof, a very complicated proof, was certified by a computer. So, the computer is helping, even if it's not really AI-based.

All in all, the real moment will come when machines will propose a new proof, or a proof of a theorem, that has not yet been proven. That would be a real breakthrough for me. For now, what do algorithms actually do? They just look at many, many papers on the web and, without understanding, imitate what they find in them. This is already something, there is clearly intelligence at play here. The big breakthrough will happen when the machines will, as I said, suggest a new proof, make a connection between two areas of mathematics, or suggest a new way of looking at a problem. These things are typically human. I haven't seen this in a machine before.

I assume this is the same with literature, with art… When the machines will propose something that we have never seen before, then we, the mathematicians, will be in danger somehow (laughs), but for now I'm not worried.

**Echterhölter:** How are these new AI applications different from once successful software like Mathematica and the computer as a numerical tool?

**Biau:** Software such as Mathematica or MAPLE, which have been used for a long time by mathematicians, are very different in that they perform complicated calculations and operations as directed by the operator, i.e. the mathematician. They are therefore very valuable tools to help mathematicians perform difficult calculations and simplify results. However, they work very differently than the algorithms that I mentioned above, which use data to propose new results to mathematicians. Eventually, of course, all these tools will converge.

**Echterhölter:** Would you say that some groups within mathematics are more open towards using these new tools and turning to the guidance of machines?

**Biau:** The question of how AI is changing the way we think is very interesting. One way of looking at this would be to observe how students, the young generation, behave. I have seen a major change with my students in mathematics, graduate and PhD students. The way they do mathematics today is entirely different from the way my students did it 10, 15 years ago. Now, they are fully integrating and utilizing new tools, for example to compute a series, or to prove that a function has a given property… We do not even attempt to prove it with

mathematical arguments, we just trust the computer. It is a new way of learning mathematics, which fully integrates the machine as part of the process.

**Echterhölter:** You served as the president of the French Statistical Society (Société française de statistique) from 2015 to 2018. This was a time during which programming libraries for AI multiplied and the public got an idea about what was going on in the aftermath of Alpha Go's win over a human at the traditional board game. Deep learning produced its first staggering results, although it had been around much longer. This success entailed a shift in the underlying statistical approaches. One general transformation seems to have been from Markovian models to convolutional neural networks and mass data approaches. Given that AI has a statistical anatomy, how did the statistics community react to this new heyday of AI after its 30 years of winter? Did AI have immediate adversaries among statisticians?

**Biau:** Statistics today finds itself in a rather paradoxical situation. On the one hand, it is indispensable for the understanding, analysis and implementation of modern machine learning methods, which are all based on data and therefore on techniques involving the science of randomness. On the other hand, the application conditions of statistics within machine learning are very different from its usual perimeter, since statistics is now confronted with models of gigantic dimensions and ever larger sample sizes. It is therefore a real challenge for statisticians today to be able to answer all these new questions! To do this, they have to adapt their tools, devise new methods and develop concepts, some of which have not changed for several centuries! But rest assured, statisticians are adapting perfectly to this new world and I am impressed by the speed at which the discipline is evolving. The younger generations of statisticians have perfectly understood the issues at stake and I have no worries about the future of the discipline.

**Echterhölter:** What is the specific relation of statistics to data, in comparison to mathematics, and does this specific relation change at all just because of AI? In the 19th century statistical societies in many countries produced and collected data, and did not just develop stochastics. One precursor to the French Statistical Society is a good testament to this rule: a founder of the "Société de statistique de Paris", Louis-René Villermé, was among the first to formulate the social question from 1860 onwards, and did so by backing up his claims about the health of workers with detailed numbers and data. Historically speaking,

this means that statistical societies were as much about observing, describing and criticizing society through numbers, as they were about developing mathematical methods. During the 20th century this clearly changed, but does the discipline of statistics have to maintain a specific relationship to data and databases?

**Biau:** This is an interesting question. While data is at the heart of statistics (indeed, etymologically, the word statistics comes from the German word "Staatenkunde" (the knowledge or science of the state), the latter has tended to evolve, around the 90s of the 20th century, somewhat away from reality, towards what is known as mathematical statistics, which encompasses the abstract study of models of inference and prediction. Interesting as it is, mathematical statistics does not really touch reality and remains in the ideal world of mathematics. But all that is changing today with the need to implement concrete and efficient methods for dealing with astronomical amounts of data. In some ways, this is a return to the roots for statistics, which must focus on its original raw material, namely data! In a way, statistics can thank AI.

**Echterhölter:** What does risk assessment for this new technology of AI look like in your research community? For instance, is this new technology a threat to some fields of mathematics? Do topics within statistics go extinct because of it? And to look beyond the ivory tower, how are hazards beyond mathematics discussed in your community?

**Biau:** Of course, we can talk about the amazing progress of machine learning in computer science and mathematics, but we could also talk about the progress of GTP-3 and other tools such as DALL-E… and how they are changing science.

Behind all this, however, there are some very important issues that need to be addressed. Ethics of course, but also sustainability and environmental issues. Consider, for example, that the training phase of GPT-3-based versions of ChatGPT emits tons of $CO_2$. This is something we should be aware of when we use these tools. The amount of energy needed for this type of algorithm is just crazy, and I'm not talking about all the energy used in the data centers! Moreover, there is also the very important question of social acceptance of AI. We are increasingly becoming slaves to algorithms, not only in our science, but in some ways in what we eat, how we drive, how we meet, the internet, etc. Is

this really what humanity wants? We have a lot of social problems in the world today and while I'm obviously no expert in sociology, I can't help but think that behind some of these problems is a widespread fear of a world that is becoming increasingly dehumanized by technology and AI. I think this is something fundamental that we need to think about seriously.

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