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Dipl.-Math. Oliver Müller,  
Hannover

## Graphical Model MAP Inference with Continuous Label Space in Computer Vision



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# **Graphical Model MAP Inference with Continuous Label Space in Computer Vision**

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This thesis deals with monocular object tracking from video sequences. The goal is to improve tracking of previously unseen non-rigid objects under severe articulations without relying on prior information such as detailed 3D models and without expensive offline training with manual annotations. The proposed framework tracks highly articulated objects by decomposing the target object into small parts and apply online tracking. Drift, which is a fundamental problem of online trackers, is reduced by incorporating image segmentation cues and by using a novel global consistency prior. Joint tracking and segmentation is formulated as a high-order probabilistic graphical model over continuous state variables. A novel inference method is proposed, called S-PBP, combining slice sampling and particle belief propagation. It is shown that slice sampling leads to fast convergence and does not rely on hyper-parameter tuning as opposed to competing approaches based on Metropolis-Hastings or heuristic samplers.

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# Abbreviations

|         |  |
|---------|--|
| 2D      | two-dimensional  |
| 3D      | three-dimensional  |
| ADMM    | alternating direction method of multipliers  |
| BP      | belief propagation   |
| DAG     | directed acyclic graph   |
| DD      | dual decomposition   |
| DPM     | deformable parts model   |
| DPMP    | diverse particle max-product   |
| FMP     | flexible mixtures-of-parts   |
| HOG     | histogram of oriented gradients  |
| KL      | Kullback-Leibler divergence  |
| LP      | linear program   |
| MAP     | maximum a posteriori   |
| MATLAB® | Matrix Laboratory, a proprietary programming language and IDE developed by MathWorks |
| MCMC    | Markov chain Monte-Carlo   |
| MH      | Metropolis-Hastings  |
| MH-PBP  | Metropolis-Hastings particle belief propagation                                      |
| MP-BP   | max-product belief propagation   |
| MRF     | Markov random field  |
| MuPAD®  | a computer algebra system bundled with MATLAB®                                       |
| OCT     | optical coherence tomography   |
| OTB     | online tracking benchmark  |
| PBP     | particle belief propagation  |
| PCP     | percentage of correct parts  |
| PGM     | probabilistic graphical model  |
| RGB     | red, green, and blue   |

|        |  |
|--------|--|
| RMSD   | root-mean-square deviation                           |
| S-PBP  | slice-sampling particle belief propagation           |
| SVM    | support vector machine                               |
| TRBP   | tree-reweighted belief propagation                   |
| VDPM   | visibility-aware deformable parts model              |
| VDPM-e | visibility-aware deformable parts model without edge |
| VOT    | visual object tracking                               |

# Symbols and Notation

## Symbols for Probability Theory

|   |   |
|---|---|
| $\mathcal{N}(x; \mu, \sigma)$           | normal distribution with mean $\mu$ and standard deviation $\sigma$ |
| $\Omega$                                | sample space  |
| $P$                                     | probability distribution  |
| $p(x)$                                  | probability density function  |
| $\mathbf{X}$                            | random variable vectors   |
| $X, Y, Z$                               | random variables  |
| $\mathcal{X}, \mathcal{Y}, \mathcal{Z}$ | state space   |
| $x$                                     | random variable values  |
| $\mathbf{x}, \mathbf{y}, \mathbf{z}$    | random variable value vectors                                       |

## Symbols for Probabilistic Graphical Models

|               |                 |
|---------------|-----------------|
| $\mathcal{C}$ | set of cliques  |
| $c$           | clique          |
| $\mathbf{d}$  | data            |
| $\mathcal{E}$ | edges           |
| $E(x)$        | energy function |
| $\mathcal{F}$ | factor vertices |
| $\mathcal{G}$ | graphical model |
| $g(\lambda)$  | dual function   |
| $k, l$        | state indices   |

|  |   |
|--|---|
| $\mathcal{L}$  | local polytope  |
| $L_s$  | number of states for discrete state space $\mathcal{X}_s$     |
| $\mathcal{L}(\{\mathbf{y}_\tau\}_\tau, \mathbf{y}, \lambda)$ | (augmented) Lagrangian function                               |
| $\lambda \in \Lambda$  | Lagrange multipliers  |
| $\mathcal{M}$  | marginal polytope   |
| $m_{t \rightarrow s}(x_s)$                                   | belief propagation message from vertex $t$ to $s$             |
| $\overline{m}_{t \rightarrow s}(x_t)$                        | pre-message from vertex $t$ to $s$                            |
| $\hat{m}_{t \rightarrow s}(x_s)$                             | approximate belief propagation message from vertex $t$ to $s$ |
| $\hat{\mu}_s(x_s)$   | approximate max-marginal function over variable $x_s$         |
| $\mu_s(x_s)$   | max-marginal function over variable $x_s$                     |
| $\mathcal{N}$  | neighborhood system   |
| $n = 1, \dots, N$  | BP/DD/ADMM iteration  |
| $\text{Pa}(s)$   | parents of vertex $s$   |
| $\mathcal{P}_s$  | particle set for vertex $s$                                   |
| $\phi_s(x_s), \phi_{s,t}(x_s, x_t), \phi_c(x_c)$             | unary potential, pairwise potential, and clique potential     |
| $\psi_s(x_s), \psi_{s,t}(x_s, x_t), \psi_c(x_c)$             | unary energy, pairwise energy, and clique energy              |
| $r, s, t$  | vertex indices  |
| $\mathcal{S}$  | visiting schedule   |
| $\tau \in \mathcal{T}$                                       | subproblems   |
| $\theta$   | parameter vector  |
| $\mathcal{V}$  | random variable vertices                                      |
| $Z$  | partition function  |

## Symbols for MCMC

|     |       |
|-----|-------|
| $A$ | slice |
|-----|-------|

|                      |   |
|----------------------|---|
| $i = 1, \dots, p$    | particle index  |
| $m = 1, \dots, M$    | Markov chain index  |
| $\mu(x)$             | MCMC target density function  |
| $\nu_0(x)$           | MCMC initial density function   |
| $\nu(x)$             | marginal probability  |
| $x_s^{(i)(m)}$       | particle at the $m$ -th MCMC iteration                                  |
| $\bar{x}_s^{(i)(m)}$ | candidate particle  |
| $q(x \mid y)$        | MCMC proposal density function  |
| $q_\sigma(x \mid y)$ | Gaussian proposal density with mean $y$ and standard deviation $\sigma$ |
| $T(x, y)$            | MCMC transition kernel  |
| $\mathcal{U}$        | auxiliary state space   |
| $\mathcal{U}(A)$     | uniform distribution over region $A$                                    |
| $u$                  | auxiliary variable  |
| $Z$                  | normalization constant  |
| $\{u^{(m)}\}_m$      | Markov chain auxiliary variables  |
| $\{x_s^{(i)}\}_i$    | particle  |
| $\{x^{(m)}\}_m$      | Markov chain variables  |

### Symbols for Part-based Object Tracking

|   |  |
|---|--|
| $E_{\text{con}}(\mathbf{p}, \mathbf{y})$  | global consistency energy                      |
| $E_{\text{DPM}}(\mathbf{p})$              | deformable parts model energy                  |
| $E_{\text{seg}}(\mathbf{y})$              | segmentation energy                            |
| $E_{\text{VDPM}}(\mathbf{p}, \mathbf{v})$ | visibility-aware deformable parts model energy |
| $\mathbb{I}[\cdot]$                       | indicator function                             |
| $\mu(\mathbf{p})$                         | patch mask function                            |

---

|                             |  |
|-----------------------------|--|
| $\mu$                       | patch mask matrix  |
| $\mathbf{p}$                | vector of object poses   |
| $\mathbf{v}$                | visibility $v_i$ for each DPM patch $i$                        |
| $\mathbf{x}$                | indicator vector over object poses                             |
| $\mathcal{X}^{\text{pose}}$ | pose state space   |
| $\mathcal{X}^{\text{seg}}$  | segmentation state space                                       |
| $\mathbf{y} \in \{0,1\}^N$  | foreground/background segmentation of an image with $N$ pixels |



## Abstract

This thesis deals with the development and improvement of maximum a posteriori (MAP) inference approaches in probabilistic graphical models (PGMs) and their application on challenging computer vision problems.

Many challenging computer vision tasks are modeled as MAP inference problems in PGMs. MAP inference is the problem of finding the most probable configuration of random variables for a given target problem in the exponentially large space of possible outcomes. PGMs are a family of powerful modeling languages which unify two fundamental concepts: uncertainty and graphical models. Many real-world phenomena can be modeled in form of probability distributions over continuous-valued random variables. A PGM is a language to model these distributions which typically involve a very large number of random variables. Conditional independence assumptions of the random variables play a key role in retrieving tractable models.

In the first part of this thesis, a general purpose framework for MAP inference in PGMs over continuous-valued random variables based on stochastic inference methods is developed. A novel approach, the slice-sampling particle belief propagation (S-PBP) algorithm, is developed which achieves more accurate and faster MAP estimates than heuristic sampling or Metropolis-Hastings sampling approaches. The proposed approach generates sample proposals from the max-marginal distributions using the slice sampling algorithm. By exploiting the message-passing nature of the applied MAP inference approach, the dependence on hyper-parameters is reduced and a significant speedup is achieved.

The second part of this thesis is dedicated to the application of the developed inference approaches to computer vision applications. Hereby, the main focus is in online tracking of articulated objects. The visual tracking of previously unseen objects in videos or video streams is a fundamental task in computer vision. A novel framework is proposed for part-based object tracking. The problem of automatic model initialization and the reduction of tracker drift by incorporating higher-order constraints and image segmentation cues to the tracker is addressed. A global consistency prior is proposed which enables inference of both part-based tracking and image segmentation in a joint probabilistic model. Experiments show that the joint formulation leads to improved image segmentation results as well as reduced drift in online object tracking.

**Keywords:** computer vision, probabilistic graphical models, MAP inference, Markov-chain Monte-Carlo, slice sampling, product slice sampling, articulated online tracking, visual object tracking, pose estimation

## Kurzfassung

Das Ziel dieser Arbeit ist die Erstellung und Verbesserung von Optimierungsverfahren zur Inferenz in probabilistischen graphischen Modellen (PGMs) und deren Anwendung auf Probleme im Bereich Computer-Vision.

Viele Computer-Vision Probleme werden heutzutage als MAP-Inferenz Probleme in PGMs behandelt. MAP-Inferenz beschreibt hierbei das Finden der wahrscheinlichsten Kombination von Werten im exponentiell wachsenden Raum möglicher Lösungen eines Problems. Probabilistische graphische Modelle sind eine Familie von Modellierungssprachen zur Beschreibung von Verbundwahrscheinlichkeiten über einer Menge von Zufallsvariablen. Hierbei werden zwei grundlegende Prinzipien miteinander vereint: Die Modellierung von Unsicherheiten und die Modellierung (bedingter) Unabhängigkeit von Zufallsvariablen mittels Knoten und Kanten in einem Graph.

Der erste Teil dieser Arbeit behandelt das Problem der MAP-Inferenz bei reellwertigen Zufallsvariablen mit Hilfe stochastischer Inferenzverfahren. Slice-sampling particle belief propagation (S-PBP) ist ein neu entwickelter Ansatz, welcher eine genauere MAP-Schätzung in kürzerer Zeit erlaubt als andere stochastische Verfahren. Eine Kernkomponente stochastischer Suchverfahren ist die Erzeugung von Stichproben im Lösungsraum. Bisherige Verfahren sind entweder heuristisch motiviert oder verwenden Vorschlagsverteilungen deren Parameter Problem-abhängig eingestellt werden müssen. Der in dieser Arbeit vorgestellte Ansatz erzeugt Stichproben direkt aus den Max-Marginal-Verteilungen des graphischen Modells mit Hilfe des Slice-Sampling Verfahrens. Durch Ausnutzung des Message-Passing Mechanismus der verwendeten Optimierungsverfahren wird die Parameter-Abhängigkeit verringert und eine Beschleunigung des Verfahrens erreicht.

Im zweiten Teil der Arbeit werden die zuvor entwickelten Inferenzverfahren auf Probleme im Bereich Computer-Vision angewandt. Das Hauptaugenmerk liegt hierbei auf der Online-Verfolgung artikulierter Objekte in Videosequenzen. Die visuelle Verfolgung zuvor unbekannter Objekte in Videos ist ein fundamentales Problem des maschinellen Sehens. Es wird ein neuer Ansatz zur Teile-basierten Objektverfolgung entwickelt. Das Problem der automatischen Modellinitialisierung und der Reduktion des Driftens in Teile-basierten Modellen wird durch die Integration von Bildsegmentierung und zusätzlicher Bedingungen höherer Ordnung behandelt. Es wird ein globaler Konsistenzterm vorgeschlagen, der die Teile-basierte Poseschätzung und die Bildsegmentierung in einem gemeinsamen probabilistischen Modell vereint. Experimente zeigen das die gemeinsame Schätzung von Pose und Segmentierung zum einen die Bildsegmentierung verbessert, als auch das Driften der geschätzten Pose effektiv verringert.

**Stichworte:** maschinelles Sehen, Probabilistische graphische Modelle, MAP-Inferenz, Markov-chain Monte-Carlo, Slice-Sampling, Produkt-Slice-Sampling, Artikulierte Objektverfolgung, Visuelle Objektverfolgung, Poseschätzung