DOUBLE CROSS PLAYING DIAMONDS

UNDERSTANDING INTERACTIVITY IN/BETWEEN BIGRAPHS AND DIAMONDS

Rudolf Kaehr

1. Models of Interactivity between flows and salti[1]

"Interactivity is all there is to write about: It is the Paradox and the Horizon of Realization."

Grammatologically, the Western notational system is not offering space in itself to place sameness and otherness necessary to realise interaction/ality. Alphabetism is not prepared to challenge the dynamics of interaction directly. The Chinese writing system in its scriptural structuration is able to place complex differences into itself, necessary for the development and design of formal systems and programming languages of interaction. The challenge of interactionality to Western thinking, modeling and design interactivity has to be confronted with the decline of the scientific power of alphanumeric notational systems as media of living in a complex world.[2]

The challenge I see for media artists is not only to develop interactional media constellations but also to *intervene* between the structures and dynamics of interactional systems as international corporations, governments, military and academia force them on us.[3]

1.1 Comparison of two approaches to interactivity

This paper takes the risk to compare two fundamentally different approaches to interaction and reflection in computational systems: Milner's *bigraphs* and *diamond* theory. Milner's bigraph model and theory of interaction is highly developed, while the diamond model applied to this interactional scenario and confronted with the bigraphs model is presented here for the first time.

The Milner model is presupposing a world-view (ontology, epistemology) of homogeneity and openness. Its basic operation is composition in the sense of category theory. Composition is associative and open for infinite iterability. Milner's model is a model of interaction in a global sense but it is not thematising formally the chiastic interplay of local and global aspects of interaction. Its merit is to have developed a strict separation of topography (locality) and connectivity for a unifying theory of global and mobile interaction (ubiquitous computing) surpassing, in principle, the limits of Turing computability.

1 Thanks to Marianne Dickson, Edinburgh, for bridging the corrections and correcting the bridges of this composition.

2 Kaehr 2006a

3 Kaehr 2003a, b

In contrast, the diamond model, which is just emerging,[4] is based on an *antidromic* and *parallactic* structure of combination of events in an open/closed world of a multitude of discontextural universes. In such a pluri-versal world model, each composition is having its complementary combination. With that, iterability for diamonds is not an abstract iterativity but interwoven in the concrete situations to be thematised, and determined by iterative and accretive repetitions, involving their complementary counterparts, without a privileged conceptual initial/final object.

This leads to a theory of diamonds as a complementary interplay of categories and saltatories (jumpoids) with the main rules, globally, of complementarity and locally, of bridging. Diamonds are involving bi-objects belonging at once to categories and to saltatories, ruled by composition and saltisition (jump-operation).[5]

1.2 Interactionality as interplays between categories and saltatories

In less technical terms, the polycontextural approach of diamond theory is supporting three new features:

First, it supports the idea of irreducible multi-medial contextures and their qualitative incomparability. That is, different media like sound, video, picture, text, graphics, etc., are conceived as logically different and as organised and distributed conceptually in a heterarchical sense. To thematise media as a digital contexture is not more than to emphasise their informatical and physical aspect, which is as such a contexture, too.

Second, it supports the possibility of mapping the (outer) environment of a contexture (media) in itself, i.e., to offer an inner environment for reflectionality. Contextures, to be different from systems, have to reflect their environment into their own domain. Hence, a contexture has to be understood as being involved into interplays of inner and outer environments.

Third, it supports the possibility of simultaneously realising movements (actions) and complementary counter-movements on a basic level of conceptualisation and formalisation. If composition of events inside a contexture, and mediation of different contextures to a compound contexture, polycontexturality, are characterised by the rules of combination, i.e., identity, commutativity and associativity, a new feature of composition is discovered by the diamond approach, which is antidromic and parallax, corresponding structurally to the otherness of the categorical system.

4 Kaehr 1996

5 Kaehr 2007a

Therefore, the questions of interactionality in a diamond framework are not primarily, how do we globally move, physically and informatically, from one topographic place to another, but how do we move by interaction from one medium to another medium of a complex knowledge space. With the appearance of the semantic web and knowledge grid[6] such developments are unavoidable. Obviously, the polycontextural diamond approach is not opting for a principally homogeneous global field of informatical and physical events but for a discontexturality of different media, situations, contexts of meaning.

The Milner Model is well based, principally, on *category theory*, the diamond model has to develop its own new formalism, risked here as a diamondisation of category theory. Hence, both theories are in a constellation, which offers a reasonable possibility for comparisons.

Because the bigraph model is based on category theory and its concept of *composition* with its abstract *iterability*, the diamond model also has to develop a distinct concept of composition (combination), one which involves a complementarity of at least two different concepts of composition, i.e. the categorical and the saltatorical, and which is opening up the operativity of an open/closed concept of iter/alterability.

Even if only metaphorically and still vague, what is common to both models is their dichotomous, dual, *complementary* and *orthogonal* approach to interaction and interactionality. The Milner model is focused on message passing, flow of informatic objects, the diamond model on agents and their reflectional/interactional activities with an emphasis on intervention.

2. Milner's bigraph model of interaction

Out of his cloud of keywords to ubiquitous computing and interactivity, Milner chooses at his Beijing 2005 performance 3 leading features: *locality*, *mobility* and *connectivity*.[7]

2.1 Locality and connectivity

Locality

"Programming the digital computer ramifies the use of space and spatial metaphor, both for writing programs and for explaining why they work. This shows up in our vocabulary: flow chart, location, send and fetch, pointer,

6 Kaehr 2004b

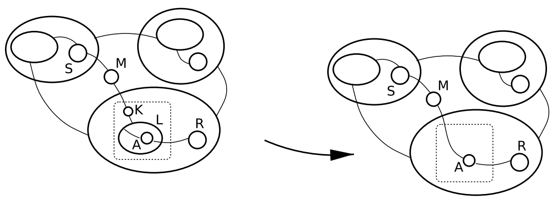
7 Milner 2005, p. 49

nesting, tree, etc. Concurrent computing expands the vocabulary further: distributed system, remote procedure call, network, routing, etc.

We are living with a striking phenomenon: the *metaphorical* space of algorithms – graph, array, and so on – is mixed with the space of *physical* reality."[8]

Physical and virtual space

"Informatic objects flow in physical space; physical objects such as mobile telephones manipulate their informatic space."

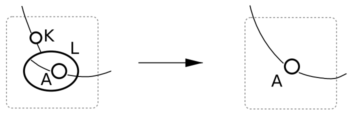


"The picture illustrates how physical and virtual space are mixed. It represents how a message M might move one step closer to its destination. The three largest nodes may represent countries, or buildings, or software agents. In each case the sender S of the message is in one, and the receiver R in another. The message is en route; the link from M back to S indicates that the messages carries the sender's address. M handles a key K that unlocks a lock L, reaching an agent A that will forward the message to R; this unlocking is represented by a reaction rule that will reconfigure the pattern in the dashed box as shown, whenever and wherever this patterns arises."[9]

"Bigraphical reactive systems are a model of information flow in which both *locality* and *connectivity* are prominent. In the graphical presentation these are seen directly; in the mathematical presentation they are the subject of a theory that uses a modest amount of algebra and category theory. A bigraph may reconfigure both its locality and its connectivity. The example pictured above shows how reconfiguration is defined by reaction rules; in that case, the rule may be pictured thus:

The mathematical structure of bigraphs allows concepts to be treated somewhat independently; for example, connectivity and locality are treated orthogonally."[10]

8 Milner 2007, p. 1
9 Milner 2007, p. 1
10 Milner 2007, p. 2



"So the challenge to bigraphs is to provide a uniform behavioural theory, allowing many process calculi to be expressed in the same frame while preserving their treatment of behaviour."[11]

The aim of a new design

"The challenge for global ubiquitous computing is to devise theories and design principles in close collaboration, ..."[12]

"The long-term aim of this work is to provide a model of computation on a *global* scale, as represented by the Internet and the World Wide Web. The aim is not just to build a mathematical model in which we can analyse systems that already exist. Beyond that, we seek a theory to *guide* the specification, design and programming of these systems, to guide future adaptations of them, and not to deteriorate when these adaptations are implemented. [...]

This will only be achieved if we can *reverse* the typical order of events, in which design and implementation come first, modelling later (or never). For example, a programming language is rarely based thoroughly upon a theoretical model. This has inevitably meant that our initial understanding of designed systems is brittle, and deteriorates seriously as they are adapted.

We believe that the only acceptable solution, in the long run, is for system designs to be expressed with the concepts and notations of a theory rich enough to admit all that the designers wish."[13]

2.2 Strategies of orthogonal simultaneity

"So our strategy here is to tackle just two aspects – *mobile connectivity* and *mobile locality* – simultaneously. In fact this combination contains a novel challenge: to what extent in a model should connectivity and locality be interdependent? In plain words, does where you are affect whom you can talk to? To a user of the Internet there is total independence, and we want to model the Internet at a high level, in the way its connectivity appears to users. But to the engineer these remote communications are not atomic, but

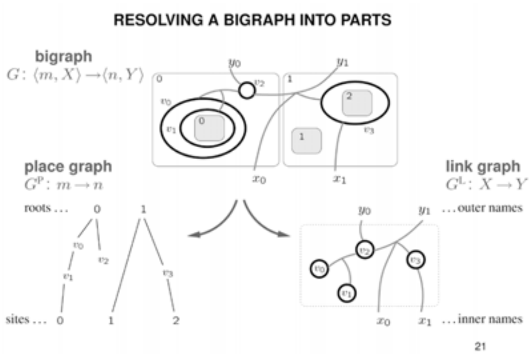
11 Milner 2007, p. 2

12 Milner 2005, p. 64

13 Milner 2004b, p. 7

represented by chains of interactions between neighbours, and we should also provide a low-level model, which rejects this reality. So we want to have it both ways; furthermore, we want to be able to describe rigorously how the high-level model is realised by the low-level one."[14]

Milner's Model of bigraphs[15]



2.2.1 Statics of interaction: Categorical framework

"Abstract. This paper axiomatises the structure of *bigraphs*, and proves that the resulting theory is *complete*. Bigraphs are graphs with double structure, representing *locality* and *connectivity*. They have been shown to represent dynamic theories for the pi-calculus, mobile ambients and Petri nets, in a way that is faithful to each of those models of discrete behaviour. While the main purpose of bigraphs is to understand mobile systems, a prerequisite for this understanding is a well-behaved theory of the structure of states in such systems. The algebra of bigraph structure is surprisingly simple, as the paper demonstrates; this is because bigraphs treat *locality* and *connectivity* *orthogonally*."[16]

2.2.2 Dynamics of interaction: Labeled process calculi

"Let us repeat: in a pure bigraph G : <m, X> –> <n, Y> we admit no association between its outer names Y and the roots (regions) n, nor between the

14 Milner 2004b, p. 7
15 Milner 2006, p. 21
16 Milner 2004a, p. 1

inner names X and the sites m. It is this dissociation that enables us to treat locality and connectivity independently, yielding a tractable theory."[17]

The dynamics of bigraphs is formalised by labeled process calculi:

"The challenge from process calculi is to provide a uniform behavioural theory, so that many process calculi can be expressed in the same frame without seriously affecting their treatment of behaviour. We now outline how research leading up to the bigraphical model has addressed this challenge.

It is common to present the dynamics of processes by means of reactions (also known as rewriting rules) of the form r –> r', meaning that r can change its state to r' in suitable contexts. In process calculi this treatment is typically refined into labelled transitions of the form a $\to^{l}$ a', where the label l is drawn from some vocabulary expressing the possible interactions between an agent *a* and its environment. These transitions have the great advantage that they support the definition of behavioural preorders and equivalences, such as traces, failures and bisimilarity. But the definition of those transitions tends to be tailored for each calculus."[18]

2.2.3 Formalisation of interaction: Bigraphs as tensor categories

"This chapter establishes place graphs, link graphs and bigraphs as arrows in certain kinds of category. Any kind of category is concerned with operations upon arrows, especially *composition*."[19]

"Note that this *combination* is quite distinct from the categorical composition used to insert one bigraph into another (e.g. an agent into a context). But it is simply related to them; to compose two bigraphs categorically, we first resolve them into their respective *place* graphs and *link* graphs, then compose these, and finally combine the results into a new bigraph."[20]

2.2.4 Axiomatics of bigraphs

"The topic of this paper is to axiomatise the resulting structure of bigraphs. The justication for such a specific topic is threefold.

First, the work already cited gives ample evidence that a graphical structure combining topography with connectivity has wide application in com-

17 Milner 2004b, p. 20

18 Milner 2005, p. 8

19 Milner 2007, p. 13

20 Milner 2004b, p. 19

puter science; for as we have seen it brings unity to at least three models of discrete dynamics, each of which has already many applications.

Second, it appears that the algebraic treatment of such dual structures has not been previously addressed; yet the behaviour of systems whose connectivity and topography are both reconfigurable may be so complex that their dynamics cannot be properly understood without a complete and rigorous treatment of their statics. Bigraphs are just one possible treatment of such dual structure, but it is likely that their static theory can be modified for other treatments.

Third, as we shall see, dual structures seem to require a novel kind of normal form which is essential to a proof of axiomatic *completeness*."[21]

Axiomatics (Table 1)

CATEGORICAL AXIOMS:

$$
\begin{array}{rcll}
A\,\mathsf{id} = & A & = \mathsf{id}\,A & \\
A(BC) & = & (AB)C & \\
A \otimes \mathsf{id}_\epsilon = & A & = \mathsf{id}_\epsilon \otimes A & \\
A \otimes (B \otimes C) & = & (A \otimes B) \otimes C & \\
(A_1 \otimes B_1)(A_0 \otimes B_0) & = & (A_1 A_0) \otimes (B_1 B_0) & \\
\gamma_{I,\epsilon} & = & \mathsf{id}_I & \\
\gamma_{J,I}\gamma_{I,J} & = & \mathsf{id}_{I \otimes J} & \\
\gamma_{I,K}(A \otimes B) & = & (B \otimes A)\gamma_{H,J} & (A\colon H \to I, B\colon J \to K)
\end{array}
$$

LINK AXIOMS:

$$
\begin{array}{rcl}
/y \circ {}^y/x & = & /x \\
/y \circ y & = & \mathsf{id}_\epsilon \\
{}^z/(Y \uplus y) \circ (\mathsf{id}_Y \otimes {}^y/X) & = & {}^z/(Y \uplus X)
\end{array}
$$

PLACE AXIOMS:

$$
\begin{array}{rcll}
merge(1 \otimes \mathsf{id}_1) & = & \mathsf{id}_1 & \text{(unit)} \\
merge(merge \otimes \mathsf{id}_1) & = & merge(\mathsf{id}_1 \otimes merge) & \text{(associative)} \\
merge\,\gamma_{1,1} & = & merge & \text{(commutative)}
\end{array}
$$

NODE AXIOMS:

$$
(\mathsf{id}_1 \otimes \alpha)\mathsf{K}_{\vec{x}} = \mathsf{K}_{\alpha(\vec{x})}\,.
$$

"In other words, the axioms are both sound and complete. They say simple things: The *place* axioms say that join is commutative, has a unit and is associative; the *link* axioms say that the formation of links obeys obvious rules; the node axiom says that we can name ports arbitrarily."[22]

2.2.5 Completeness of the axiom system

"The completeness of the axiom system in *Table 1* depends primarily on two things: first, that all linking can be exposed at the outermost level of an

21 Milner 2004a, p. 4

22 Milner 2004a, p. 23

expression; second, that we have a strict symmetric monoidal category of bigraphs, with a tensor that is partial on objects. Crucial to the tensor is that it is bifunctorial, i.e. $(A_1 \times B_1)(A_0 \times B_0) = (A_1A_0) \times (B_1B_0)$; this axiom underlies most of our manipulations. Thus the discrete normal form, DNF, has been crucial for the proof of completeness."[23]

2.3 Orthogonality of topography and connectivity

2.3.1 Underlying world model

The bigraph model of interaction is highly flexible and is liberating further research from unnecessary fixations. Bigraphical reactive (re-writing) systems as models of information flow are dealing with *locality* and *connectivity* as *orthogonal* events, distributed over two dimensions. Such a separation of structural locality and behavioural connectivity enables a clear modeling and an effective formalisation as a bigraph or bipartide system. Spaciality is conceived as static, formalised by category theory and behaviour as dynamic, formalised by process calculi (pi-calculus).

The bigraph model of interaction seems to belong to a world model with the characteristics of: *"Everything in this world is changing but the world in which everything is changing doesn't change."*[24] Ubiquitous and global computing is presupposing an epistemologically uniform, homogeneous and unique world of physical and informatical events.

Diamond theory can be set in some kind of a correspondence with a bipartide model but it is turning to a world model where there are many worlds in which things are changing and in which worlds themselves are changing too. Diamond Theory is involved not in a new super-stable world but in the game of interactionality/reflectionality between worlds and events, hence enabling system designers and media artists to *intervene* in and between those worlds guided by the metamorphic dynamics of polycontextural diamonds.

Messages in the diamond model are conceived as polycontextural and as belonging simultaneously to different contextures of irreducible kinds of meaning. Message passing in such a model is not done by the metaphor of key/lock/unlock/agent in a location/connectivity setting because a key in this pluriversal world-model appears always as necessarily polysemic and its acceptance has to be negotiated by reflectional and interactional activities. If such complex transactions are becoming stable in their usage, a reduction to the mono-contextural key-model can be introduced by reducing complexity.

23 Milner 2004a, p. 21

24 Kaehr 2007d

2.3.2 Chiastic transition metaphor

Hence, in a chiastic metaphor, we can state that statics in the bigraph model becomes dynamics in the diamond model; and dynamics becomes statics in the diamond setting because its dynamics is bracketed and moved into a multitude of process-structures wherein the dynamics of the different behavioural systems have an arena in which to act. Therefore, category theory as formalism for interaction has to be dynamised towards diamond theory. That is, category theory has to be diamondised towards a dynamic structural formalism, which is an operational structuration.

2.3.3 Opting for an interventional design

The British Grand Challenge project for computing is not touching the principle hierarchy between mathematics and informatics. Since the Greeks time has changed and a reversion and displacement of this hierarchy might be the grand challenge of a new understanding of global computing.[25]

From a model of interactions to a design of interactionality, the transitions to be risked might be:

From the *global*, ubiquitous and universal web of computation, to the kenomic grid of *pluriversal* contexturality, containing the chiasm of global/local scenarios.

From the *locality* in the Actor model of informatical events to the *positionality* of contextures in the kenomic grid, positioning informatic localities.

From the *mobility* in the Actor model of informatical flows between ambients (context, locality) of the same contextural (ontological, logical, semiotic) structure to a *metamorphosis* between contextures, augmenting complexity/complication of contextural scenarios implementing clusters of informatical ambients and mobility.

From the *operations* between actional ambients to the *operationality* in polycontextural situations realised by the *super-operators* (identity, replication, permutation, reduction, bifurcation) placing ambient operations into the grid.

From the *connectivity* of actions at a locality of message passing, using a key to unlock a lock of an agent, to different kinds of *mediation* between contextures containing informatical connectivity.

25 Kaehr 2003a

These transitions seem to record a catalogue of *minimal* conditions to be fulfilled to realise interactionality/reflectionality and interventionality in such complex constellations as the emerging knowledge grid.[26]

3. Diamond theory of interactionality

3.1 Diamond Strategy

Encounter

Diamond strategies are sketching transitions from the mail model of interaction in bigraphs to the *encounter* model of interactionality/reflectionality and intervention.

Before we can play the bipartide game of locking and unlocking (by passing a key in a structure of orthogonal locality and connectivity to reach an agent capable of passing the message to another agent), the *otherness* of the actors involved has to be acknowledged and accepted by all the interactional activities of the actors involved.

It can be described as the action of addressing an addressee, which is able to accept the addressing by offering its own addressable structure. After having been addressed and having the addressing accepted by the addressed and after the addresser has recognised the acceptance of being addressed and the addressing is thus established, information can be exchanged between agents in the sense of communication.[27]

Interactivity in the encounter-model, therefore, is conceived as a *mutual* action of *acceptance* and *rejection* between different agents. Only on the basis of this interactional agreement can information exchange happen.[28]

Therefore, the structure of interaction is always complex: at once realising the addresser and the inner environment of the addressee. This simultaneity of inner and outer environments of agents involves a kind of structural bifurcation and mutual actions of *acceptance* and/or *rejection* of the involved agents based on the complexity of their architectonics. That is, the addressee has to give space (*einräumen*) to the addresser to be addressed. To address and to accept to be addressed is a *mutual* action of at least two agents in a common co-created environment. Hence, the actional structure of interactionality is not only bipartide but antidromic, too. This phenomenon forces a

26 Kaehr 2006b

27 Kaehr 2004a

28 Kaehr 2004a

formalisation paradigm beyond mathematical category theory, which finds a very first attempt to a realisation in the proposed diamond theory.[29]

Intervention

An interaction of an agent, including reflections on the behaviour of a partner agent, which is intended to change the meta-rules of the partner-agent can be called an *intervention*. An agent is intervening into an interaction in attempting to change the meta-rules of the agent. An intervention takes place if an agent is interacting with another agent in a way that the agent is forced to change his meta-rules to stay in the game of computation and interaction.[30]

> The aim is not just to build a mathematical model in which we can analyse systems that already exist. Beyond that, we seek a theory to guide the specification, design and programming of these systems, to guide future adaptations of them, and not to deteriorate when these adaptations are implemented. There is much talk of the vanishing ubiquitous computer of the future, which will obtrude less and less visibly in our lives, but will pervade them more and more. Technology will enable us to create this. To speak crudely, we must make sure that we understand it before it *vanishes*.[31]

Diamond strategies are not only asking for an understanding of such trends, like the vanishing of computational challenges for users by ubiquitous computing, but for the possibility of intervention by computer designers, scientists and users into such trends. Thus, opening up interplays between users and general computation, avoiding any kind of regression into euphoria, criticism and luddism of humanistic self-defence.

3.2 Towards Diamond Theory

3.2.1 From categorical composition of morphisms to diamonds

Actions from A to B can be considered as morphisms, symbolised by an arrow from A to B, A –> B. In this sense, morphisms are universal, they occur everywhere. But morphisms don't occur in isolation, they are composed together in interesting complexions. The composition of morphisms (arrows) is defined by the coincidence of codomain (cod) and domain (dom) of the morphisms to be composed, called the matching conditions (MC). That is, (f, g)

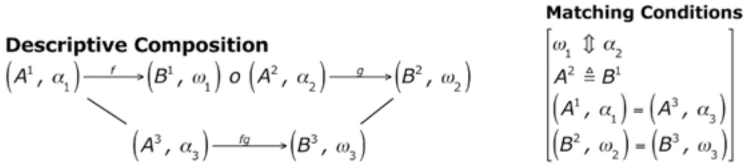
29 Kaehr 2007c

30 Kaehr 2005, 2006c

31 Milner 2004b, p. 7

is composed (f o g) iff cod(f) = dom(g). This highly general notion of morphism and composition of morphisms is studied in *Category Theory*.[32]

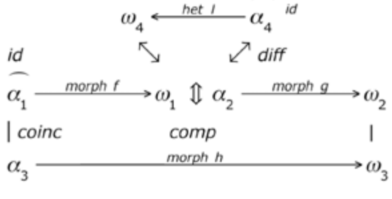
A general descriptive explication of the concept of composition of morphisms is given by the following diagram. It contains the table of the matching conditions. Here, the distinction between objects, A, B as domain and codomain properties of morphisms, and the alpha (α) and omega (ω) functionality of morphisms are included.



Hence, not only the codomain B^1 and the domain A^2 as objects have to coincide, but also the actional domain "alpha$_2$" (α_2) and the actional codomain "omega$_1$" (ω_1) as functional properties of the morphisms f and g, have to match. Obviously, the commutativity of the diagram has to fulfill, additionally, the matching conditions for (A^1, α_1) with (A^3, α_3) and (B^2, ω_2) with (B^3, ω_3), defining the composition (f o g).

First, without the actional alpha/omega-notation we get the matching conditions, coincidences, for categorical composition based on the objectional distinction of domains and codomains.

Second, stripped off of the set-theoretical or objectional content of the domains and codomains of morphism, the functionality of beginnings (α) and endings (ω) remain. Composition then means an *exchange* relation between the ending of a morphism and the beginning of another morphism, i.e., between (ω_1) and (α_2). Both founded in the coincidence relation between the actional domain of the first and the actional codomain of the second morphism, establishing the commutativity of "object-free" categorical composition, i.e., the morphism between (α_3) and (ω_3), i.e., (α_3) –> (ω_3).



32 Kaehr 2007a

Such a chiastic approach, emphasising the pure functionality of composition uncovers the possibility of a new relationship involved in the definition of *actional* composition: the complementarity of the commutative morphism between the beginning (α2) and the ending (ω1) involved in the categorical composition, building the *"antidromic and parallax"* hetero-morphism between (α4) and (ω4), i.e., (α4) –> (ω4).

Hence, functional composition of morphisms, which are represented by *order* relations, is based on the functional matching conditions, MC, of two types of relations: *exchange* and *coincidence* relation building together with the order relations, a *chiastic* pattern in form of a *diamond*. Obviously, this singular diamond is occupying a place and is localised in a grid of diamonds and thus ready to be disseminated.

Third, both thematisations together, the objectional and the actional, with morphisms and hetero-morphisms, define the diamond composition of morphisms.

3.2.2 Diamond model of system/environment

Some wordings to the diamond system/environment relationship might be listed:

What's my environment is your system.

What's your environment is my system.

What's both at once, my-system and your-system, is our-system.

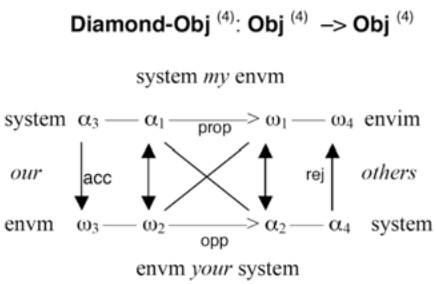
What's both at once, my-environment and your-environment, is our-environment.

What are our-environments and our-systems is the environment of others-system.

What's our-system is the environment of others-system.

What's neither my-system nor your-system is others-system.

What's neither my-environment nor your-environment is others-environment.



Some environement / system formulas

$+$: *and*, $|$: *neither – nor*, $\neg$: *not*.

$$my(syst,\ env) = your(env,\ syst)$$
$$our(syst,\ env) = others(env,\ syst)$$
$$my(syst,\ env) + your(syst,\ env) = our(syst,\ env)$$
$$my(syst,\ env)\ |\ your(syst,\ env) = others(syst,\ env)$$
$$our(env + syst) = others(env)$$
$$others(env + syst) = our(syst)$$
$$our(env\ |\ syst) = others(syst)$$
$$others(env\ |\ syst) = our(syst)$$
$$my(syst) = \neg(your(syst), others(syst))$$
$$your(syst) = \neg(my(syst), others(syst))$$
$$our(syst) = \neg(my(syst),\ your(syst), others(syst))$$
$$others(syst) = \neg(my(syst),\ your(syst), our(syst))$$

The diamond modeling of the otherness of the others incorporates the otherness into its own system. An external modeling of the others would have to put them into a different additional contexture. With that, the otherness would be secondary to the system/environment complexion under consideration. The diamond modeling is accepting the otherness of others as a *"first-class object"*, and as belonging genuinely to the complexion as such.

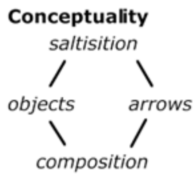
In another setting, without the "anthropomorphic" metaphors, we are distinguishing between a system and its internal and its external environment. The external environment corresponds to the *rejectional* part, the internal to the *acceptional* part of the diamond. Applied to the diamond scheme of diamondised morphisms we are directly getting the *diamond system scheme* out of the diamond-object model.

Much work has been done on interactionality/reflectionality and interventionality/interlocutionality on the basis of *polycontextural* notions and formalisms.[33] Despite its chiastic and proemial approach, this work did not yet include the others-system of the diamond model.

33 Kaehr 2005, 2006d

3.3 Diamond Structuration

Diamonds in this sketch are conceived as interplays between categories and saltatories based on morphisms and hetero-morphisms with their compositions, saltisitions and bridgings. Saltatories are the complementary concept of categories.



The conceptuality of diamond theory is introduced by an application of the *diamond strategies* to the basic concepts of category theory: *objects* and *morphisms* (arrows). Objects are understood in this setting as propositions, arrows as oppositions. Compositions appear as the both-at-once of objects and arrows, and saltisitions as the neither-nor of objects and arrows. Composition and saltisitions, hence, are complementary concepts.

saltisition, saltatory

salto mortale: jump from the apriori to the empirical (Immanuel Kant).

diamond strategies: double salto mortale from the theoretical to the hyper-theoretical.

Diamond Theory

Category $: \mathbb{A} = \left(Obj^{\mathbb{A}}, hom, id, o\right)$

Saltatory $: \mathbf{a} = \left(Obj^{a}, het, id, \|\right)$

DTh $= \left(\left[\mathbb{A}; \mathbf{a}\right], compl, diff, \bullet\right)$

Diamond duality

Category		**Saltatory**	
Cat	Cat^{op}	*Salt*	*Sa*

Categories are dealing with *composition* of morphisms and their laws. Saltatories are dealing with the jump-operation (*saltisitions*) of hetero-morphisms and their laws. Diamonds are dealing with the *interplay* of categories and saltatories. Their operation is interaction realised by the *bridging* operations.

The laws of *identity* and *associativity* are ruling compositions, as well as saltisitions. Complementarity between categories and saltatories, i.e., between acceptional and rejectional domains of diamonds, are ruled by *difference* operations. Duality operations are applicable to both, categories and saltatories.

Commutativity and associativity

Commutativity Condition

If $f, g \in MC,\ l \in MC:$

then

$$g \Diamond f = (g \circ f) \parallel l$$

$$\text{with} \begin{pmatrix} \omega(f) \Updownarrow \alpha(g) \\ diff(\alpha(g)) = \alpha(l) \\ diff(\omega(f)) = \omega(l) \end{pmatrix}$$

such that

$$\left.\begin{array}{ccc} A & \xrightarrow{f} & B \\ & {}_{gf}\searrow & \downarrow g \\ & & C \end{array}\right\| \; b_1 \xleftarrow{\;l\;} b_2$$

bi – commutes.

Associativity Condition

If $f, g, h, k \in MC$ *and* $l, m, n \in MC:$

then

$$(k \Diamond h \Diamond g \Diamond f) =$$

$$\left[\begin{array}{l|l} k \circ (h \circ (g \circ f)) = k \circ ((h \circ g) \circ f) & \\ ((k \circ h) \circ g) \circ f = (k \circ (h \circ g)) \circ f & l \parallel (m \parallel n) = (l \parallel m) \parallel n \\ (k \circ h) \circ (g \circ f) = k \circ (h \circ g) \circ f & \end{array}\right]$$

3.3.1 Identity and difference

"This shift becomes even more apparent if one examines the foundational concepts Nishida develops later in his career, the '*self-identity of the absolute contradiction*' and the '*many in one, one in many*' (tasokuitsu, issokuta); the former can be paraphrased as the 'identity of absolute difference' and the latter as 'plurality in oneness, oneness in plurality'."[34]

34 Kopf 2004, p. 80

Identity and difference morphisms

$$\textbf{bi-Object}\left[\mathbf{X},\ \mathbf{x}\right]$$

$$\left.\begin{array}{l} \widehat{id} \\ x \in Salt \\ \updownarrow diff \\ X \in Cat \\ \widehat{id} \end{array}\right\} \in Diam$$

Identity is a mapping onto-itself as itself.

For each object X of a category an identity morphism, $ID_{[X,\ X]}$, which has domain X in the category and codomain X in the same category exists. Called ID_X or id_X for $ID_{[X,\ X]}$.

For each object x of a saltatory an identity morphism, $ID_{[x,\ x]}$, which has domain x in the saltatory and codomain x in the same saltatory exists. Called ID_x or id_x for $ID_{[x,\ x]}$.

Difference is a mapping onto-itself as other.

For each object X of a category a difference-morphism $DIFF_{[X,\ x]}$, which has domain X in the category and codomain x in the saltatory exists.

For each object x of a saltatory a difference morphism, $DIFF_{[x,\ X]}$, which has domain x in the saltatory and codomain X in the category exists.

This wording is a strict paraphrase of the common wordings of category theory. It also emulates its architectonics: from objects to morphisms to isomorphisms and to natural transformation, etc. Nevertheless it is not yet reflecting the reversed architectonics of the diamond way of thinking, where objects occur last and not first.

Identity and difference composition

ID and DIFF composition

Identity	**Difference**
$\forall f,\ X,\ Y,\ o \in Cat:$	$Om\ Cat,\ Salt \in Diam:$
$f\ o_{XXY}\ ID_X = f = ID_Y\ o_{XYY}\ f.$	$\forall [X,\ x],\ [Y,\ y] \in Diam$
$\forall l,\ x,\ y,\ \parallel \in Salt:$	$[f,\ l]\ (o,\ \parallel)_{[XYy,\ xyY]}\ DIFF_{[Y,\ y]}$
$l\ \parallel_{xxy}\ ID_x = l = ID_y\ \parallel_{xyy}\ l.$	$= [f,\ l] =$
	$DIFF_{[y,\ Y]}\ (\parallel,\ o)_{[xyY,\ XYy]}\ [l,\ f].$

3.3.2 Diamond concepts between iso- and xenomorphism

> *"One philosophical reason for categorification is that it refines our concept of 'sameness' by allowing us to distinguish between isomorphism and equality."*[35]

Isomorphisms

Isomorphism in Cat : $\mathbf{Cat_{Iso}}$

$$\forall f,\ g \in Cat:$$

$$X \underset{g}{\overset{f}{\rightleftarrows}} Y \ iff \ \begin{cases} g \circ f = id_X \\ f \circ g = id_Y . \end{cases}$$

Isomorphism in Salt : $\mathbf{Salt_{Iso}}$

$$\forall l,\ m \in Salt:$$

$$x \underset{m}{\overset{l}{\rightleftarrows}} y \ iff \ \begin{cases} m \parallel l = id_x \\ l \parallel m = id_y . \end{cases}$$

Diamond Isomorphism : $\mathbf{Diam_{Iso}}$

$$Om\ Cat,\ Salt \in Diam:$$

right - domain - ISO :

$$\begin{pmatrix} \hat{x} \\ \updownarrow diff \\ X \underset{g}{\overset{f}{\rightleftarrows}} Y \end{pmatrix} iff \begin{bmatrix} (g \circ f) \bullet id_x = id_{[X,x]} \\ id_x \bullet (f \circ g) = id_{[Y,x]} \end{bmatrix}$$

left - codomain - ISO :

$$\begin{pmatrix} & \hat{y} \\ & \updownarrow diff \\ X \underset{g}{\overset{f}{\rightleftarrows}} Y & \end{pmatrix} iff \begin{bmatrix} (g \circ f) \bullet id_y = id_{[X,y]} \\ id_y \bullet (f \circ g) = id_{[Y,y]} \end{bmatrix}$$

Hetero - ISO

right - domain - ISO :

$$\begin{pmatrix} x \underset{m}{\overset{l}{\rightleftarrows}} y \\ \updownarrow diff \\ \hat{X} \end{pmatrix} iff \begin{bmatrix} (l \parallel m) \bullet id_X = id_{[x,X]} \\ id_X \bullet (m \parallel l) = id_{[X,y]} \end{bmatrix}$$

left - codomain - ISO :

$$\begin{pmatrix} x \underset{m}{\overset{l}{\rightleftarrows}} y & \\ & \updownarrow diff \\ & \hat{Y} \end{pmatrix} iff \begin{bmatrix} (m \parallel l) \bullet id_Y = id_{[y,Y]} \\ id_Y \bullet (l \parallel m) = id_{[Y,y]} \end{bmatrix}$$

Category theory is studying, at first, isomorphisms between objects as domains and codomains of morphisms, then the trip goes on with functors, natural transformations and so on. Their basic element, thus, is an elementary, single morphism and their basic operation is a single identity morphism. Diamond theory is dealing with the interplay between categories and saltatories, hence, the elementary situation is not a single morphism but the interaction of the selected morphism and its two corresponding, i.e., interacting hetero-morphisms based on identity and difference operations. That is, the

35 Baez/Dolan 1998, p. 7

domain and the codomain of the selected morphism has to consider the corresponding domain and codomain of the hetero-morphisms involved. This is ruled by the difference operation.

Hence, the isolated objects as domains and codomains have to be supplemented by their own counter-parts, codomain and domain, to build their hetero-morphisms. In other words, the full interplay of morphisms, identity and difference mappings, has to be involved to realise proper diamond iso- and xenomorphisms.

Full *combined* isomorphisms between morphisms and hetero-morphisms are naturally constructed out of the partial iso- and xenomorphisms.[36]

3.3.3 Diamond concept of transversality

A difference-philosophical interpretation of *transversal* isomorphisms could be found in the classical formulations of *"The identity of oppositions, i.e., the identity of difference and identity."* and *"The difference of identity and difference"*. Both formulations are in some sense dual.

Further, more complex isomorphisms are easily composed by a combination of right- and left-isomorphisms.

Transversality ISO

$$trsv_A : diff(A) \longrightarrow (B)$$
$$trsv_B : A \longrightarrow diff(B).$$

right - transversal - ISO

$$\left(\begin{array}{ccc} \widehat{X} & & \\ \updownarrow diff & \nwarrow\!\searrow trsv & \\ X & \underset{g}{\overset{f}{\rightleftarrows}} & Y \end{array}\right) \; iff \; \left[\begin{array}{l} trsv_{[X,Y]} \bullet diff_{[X,X]} = id_{[Y]} \\ trsv_{[X,Y]} \bullet (f \circ g) = diff_{[Y,X]} \end{array}\right.$$

left - transversal - ISO :

$$\left(\begin{array}{ccc} & & \widehat{Y} \\ & \swarrow\!\nearrow trsv & \updownarrow diff \\ X & \underset{g}{\overset{f}{\rightleftarrows}} & Y \end{array}\right) \; iff \; \left[\begin{array}{l} (g \circ f) \bullet trsv_{[Y,X]} = diff_{[Y,X]} \\ trsv_{[X,Y]} \bullet diff_{[Y,Y]} = id_{[X]} \end{array}\right.$$

3.3.4 Facets of diamond isomorphisms

The concept of diamond isomorphisms is not solely dynamising the realm of sameness, as is the aim of category theory, but it is also inter-wined with the differentness and strangeness of otherness.[37]

36 Kaehr 2007a

37 Kaehr 2008a

1. Sameness (up to isomorphism)

$\forall f, g, X, Y \in Cat:$

$$\widehat{X} \underset{g}{\overset{f}{\rightleftarrows}} \widehat{Y} \text{ iff } \begin{bmatrix} g \circ f = id_{[X,X]} \\ f \circ g = id_{[Y,Y]} \end{bmatrix}$$

2. Differentness (up to xenomorphism)

$\forall f, X, Y \in Cat, \forall l, x, y \in Salt:$

$$\begin{pmatrix} & x \overset{l}{\longleftarrow} y & \\ diff \updownarrow & & \updownarrow diff \\ & X \overset{f}{\longrightarrow} Y & \end{pmatrix} \text{ iff } \begin{bmatrix} l \bullet f = diff_{[X,x]} \\ f \bullet l = diff_{[y,Y]} \end{bmatrix}$$

3. Strangeness (up to heteromorphism)

$\forall f, g, X, Y \in Cat, \forall l, m, x, y \in Salt:$

$$\begin{pmatrix} & x \underset{m}{\overset{l}{\rightleftarrows}} y \\ & \updownarrow diff \\ X \underset{g}{\overset{f}{\rightleftarrows}} Y & \end{pmatrix} \text{ iff } \begin{bmatrix} (g \circ f) \bullet (m \parallel l) = id_{[X,x]} \\ (f \circ g) \bullet (l \parallel m) = id_{[Y,y]} \end{bmatrix}$$

3.4 Interactionality as interplays in diamonds

Interactionality of diamonds studies the interaction between disseminated categories and saltatories of polycontextural diamond systems. Given contextures in isolation, topics like *duality* and *complementarity* in diamonds are interactional, but they do not yet considering the inter-twining and intervening properties of interactivity as it happens with *bridging*. Thus, interactionality as an intra-contextural interplay occurs in elementary diamonds in forms of *duality*, *complementarity*, *bridging* and *distributivity*.

Duality for Categories: *"two for the price of one"*

> The Duality Principle for Categories states
>
> *Whenever a property P holds for all categories,*
> *then the property P^{op} holds for all categories.*
>
> The proof of this (extremely useful) principle follows immediately from the facts that for all categories A and properties P
>
> (1) $(\mathbf{A}^{op})^{op} = \mathbf{A}$, and
>
> (2) $P^{op}(\mathbf{A})$ holds if and only if $P(\mathbf{A}^{op})$ holds.[38]

Duality is defined for diamonds as duality of categories and duality of saltatories.

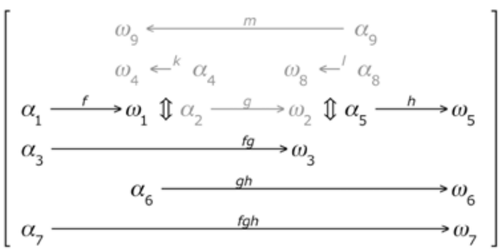
38 Adamek/Herrlich 2004, p. 27

Complementarity of formal languages

> The general principle underlying these limitations was called the *linguistic complementarity* by Loefgren. It states that in no language (i.e. a system for generating expressions with a specific meaning) can the process of interpretation of the expressions be completely described *within* the language itself. In other words, the procedure for determining the meaning of expressions must involve entities from outside the language, i.e. from what we have called the context. The reason is simply that the terms of a language are finite and changeless, whereas their possible interpretations are infinite and changing.[39]

The double meaning of diamond objects, bi-objects, is complementary and in their orientations they are not parallel but *antidromic* and *deferred* regarding the complementary system.

Bridging categories and saltatories



Bridging is not an operation of mediation or switching of and between diamonds or acceptional and rejectional actions in diamonds, but an operation to knot the two realms together, the categorical and the saltatorical. In the diagram, between the hetero-morphism *k*, *l*, the morphism *g* is offering a bridge, marked in red, and thus interacting between the saltatorical and the categorical domain of the diamond. Complementarily, the two bridge pillars of the bridge are offered by the two hetero-morphisms *l*, *k* defining the bridgework *g*. Thus, bridge and bridging are complementary actions, too. Both are reflecting the complementarity between categories and saltatories.

Distributivity of composition, saltisition and bridging

Because diamonds are based on interplays between categories and saltatories, which are involved with two fundamental operations: composition (o) and saltisition (| |) with bridging (•) too, it is reasonable to find interactive

39 Heylighen: § 6.3

laws as distributivity between those basic operators inside the very definition of the conception of diamonds.

3.4.1 Duality in diamonds as duality in categories and saltatories

Duality in Diamonds

duality in categories	*duality in saltatories*
$(g\ o\ f) = A \to C$	
$dual(g\ o\ f) = dual\ (dual(B \to C)\ o\ dual(A \to B))$	$u = (\omega_4 \leftarrow \alpha_4) = compl(g\ o\ f)$
$= dual\ ((B \leftarrow C)\ o\ (A \leftarrow B))$	$dual(compl(g\ o\ f)) = dual(u)$
$= ((A \leftarrow B)\ o\ (B \leftarrow C))$	$dual(u) = dual(\omega_4 \leftarrow \alpha_4)$
$= (A \leftarrow B \leftarrow C)$	$= (\alpha_4 \to \omega_4)$.
$= A \leftarrow C.$	$compl(dual(g\ o\ f)) = compl(f\ o\ g) = (\alpha_4 \to \omega_4)$.
Hence, $((g\ o\ f) = A \to C) \in Cat$	*Hence,* $(u = (\omega_4 \leftarrow \alpha_4)) \in Salt$
iff	*iff*
$(dual(g\ o\ f) = A \leftarrow C) \in Cat^{op}$.	$(dual(u) = \alpha_4 \to \omega_4) \in Salt^{op}$.
$X = g \lozenge f = [(g\ o\ f); u]$:	
$X \in Cat$ *iff* $dual(X) \in Cat^{op}$	$X \in Salt$ *iff* $dual(X) \in Salt^{op}$

3.4.2 Complementarity of categories and saltatories

Complementarity of Acc and Rej

$X \in Acc$ *iff* $compl(X) \in Rej$

$X = g\ o\ f$:

1. $X \in Acc$ *if* $compl(X) \in Rej$

$$compl(g\ o\ f) = compl\ (compl(g)\ o\ compl(f))$$
$$= compl(diff\ (cod(f))\ o\ diff\ (dom(g)))$$
$$= compl((\overline{B_{cod}})\ o\ (\overline{B_{dom}})) = \omega_4 \leftarrow \alpha_4.$$

$(u : \omega_4 \leftarrow \alpha_4) \in Rej$

Hence, $(g\ o\ f) \in Acc$ *if* $(u : \omega_4 \leftarrow \alpha_4) \in Rej$

$(g\ o\ f) \in Acc$ *if* $(\overline{g\ o\ f}) \in Rej$.

2. $compl(X) \in Rej$ *if* $X \in Acc$

$$compl(\omega_4 \leftarrow \alpha_4) = compl(compl(\omega_4) \leftarrow compl(\alpha_4))$$
$$= compl((A_{dom} \to B_{cod}) \leftarrow (B_{dom} \to C_{cod}))$$
$$= ((A_{dom} \to B_{cod})\ o\ (B_{dom} \to C_{cod}))$$
$$= (f\ o\ g).$$

3. *Hence,* $X \in Acc$ *iff* $compl(X) \in Rej$.

3.4.3 Bridging between categories and saltatories

This new feature of bridge/bridging is ruling concrete intrinsic interactions.

Bridging conditions and associativity for interactions

Bridge and Bridging Conditions BC

1. $\forall k, l, n \in HET, \forall f, g, h \in MORPH$:

a. composition

$g \circ f, g \circ h,$

$(h \circ g) \circ f, h \circ (g \circ f) \in MC,$

b. saltisition

$l \parallel k, n \parallel l,$

$n \parallel (l \parallel k), (n \parallel l) \parallel k \in \overline{MC},$

c. bridges

$g \perp k, l \perp g,$

$(l \perp g) \perp k, l \perp (g \perp k)$ are in $\widehat{BC}$.

d. bridging

$g \bullet k, l \bullet g,$

$(l \bullet g) \bullet k, l \bullet (g \bullet k)$ are in BC.

2. $(g \bullet k) \in BC$ iff $dom(k) = diff(dom(g))$,

$(l \bullet g) \in BC$ iff $cod(l) = diff(cod(g))$,

$(l \bullet g \bullet k) \in BC$ iff $(g \bullet k), (l \bullet g) \in BC$.

3. $(g \perp k) \in \widehat{BC}$ iff $diff(dom(k)) = dom(g)$,

$(l \perp g) \in \widehat{BC}$ iff $diff(cod(l)) = cod(g)$,

$(l \perp g \perp k) \in \widehat{BC}$ iff $(g \perp k), (l \perp g) \in \widehat{BC}$.

Bridging

Assoziativity :

If $k, g, l \in BC$, then $(k \bullet g) \bullet l = k \bullet (g \bullet l)$,

Bridging :

$bridging_{(g,l,k)} : het(\omega_4, \alpha_4) \bullet hom(\alpha_2, \omega_2) \bullet het(\omega_8, \alpha_8) \rightarrow het(\omega_9, \alpha_9)$.

Bridge

Assoziativity :

If $k, g, l \in \widehat{BC}$, then $(k \perp g) \perp l = k \perp (g \perp l)$,

Bridge :

$bridge_{(g,l,k)} : het(\omega_4, \alpha_4) \perp hom(\alpha_2, \omega_2) \perp het(\omega_8, \alpha_8) \rightarrow het(\omega_9, \alpha_9)$.

Bridges vs. Bridging vs. Jumping

$(l \perp g \perp k) \triangleq (l \bullet g \bullet k) \triangleq (l \parallel k),$

$(l \perp g \bullet k) \triangleq (l \bullet g \perp k) \triangleq (l \parallel k),$

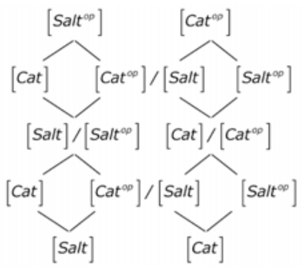
$(l \bullet g \perp k) \triangleq (l \perp g \bullet k) \triangleq (l \parallel k).$

$diff(\perp) = (\bullet), (\perp) = diff(\bullet).$

4. Bigraphs in diamond webs

Instead of labelling transitions of the behavioural calculus, the whole system of bigraphs could be labeled (disseminated), i.e., distributed and mediated. Reflectionality between disseminated bigraphs, then might be realised by the "double-character" of diamonds. The possibility to disseminate bigraphs would open up a chiastic chain of connectivity and locality graphs, of statics and dynamics, as a new play of interactionality/reflectionality between bigraphical systems.

4.1 Disseminated Diamonds



Diamonds, in this possible dissemination, are mapped as categories and saltatories with their dualities.

Mediation between diamonds happens *horizontally*, by complementarity and accretion from dual-categories to saltatories. And *vertically*, by duality and iteration from one diamond to another diamond of the grid.[40]

4.2 Towards a diamond web of bigraphs

In this setting we would have to introduce first the dual theory of bigraphs, which are themselves incorporating the dual structure of topography and connectivity. The more intriguing step would be to develop the complementary system to bigraphs and its duality, placed in saltatories. Both together are building the diamond of bigraphs, which then could be disseminated to model and design interactionality and reflectionality in a polycontextural system of interaction including the chiasm of global and local situations. Such a diamond web would not be restricted to informatic and physical global

40 Kaehr 2007c

interactions like bigraphs but would be open to offer a framework for knowledge related semantic and pragmatic aspects of pluriversal computation and communication. Dissemination of diamonds might offer a scheme for a distribution and mediation of the orthogonality of connectivity and locality in bigraphs, which are themselves thematised as dualities.[41]

From a more futuristic vision, also with not much theory, Hai Zhuge (Beijing) develops the idea and sketches some steps towards a methodology of a *knowledge grid*, which is to *"foster worldwide knowledge creation, evolution, inheritance, and sharing in a world of humans, roles and machines."*[42]

41 Kaehr 2008b

42 Zhuge 2004, p. 1; see also Kaehr 2007e, f

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