Mind

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Consciousness and Psychometric Modeling

Entering ›consciousness‹ into Google Scholar on Dec. 16th 2020 yielded 4.500.000 hits. There is a plethora of researches, paradigms, and results related to consciousness. One might conclude that this is an especially productive and innovative area of research. However, the wealth of issues, results, and paradigms on specific conscious processes as well as on general issues of consciousness may also indicate that some scientific disciplines (e.g., neuroscience, theology, philosophy, psychology) are partly unable to cope with the broadness, heterogeneity, and complexities implied by the topics related to ›consciousness‹. It seems that research on consciousness is still quite a challenge. In the following, a transdisciplinary perspective on consciousness will be tried out. The transdisciplinary perspective is based on the analysis of a possible parallelism between (1) some neurocognitive results on consciousness, (2) some philosophical accounts on consciousness, and (3) psychometric modeling and its possible relationship to consciousness. The arguments from each discipline are outlined in a separate section below. The parallelism that will be outlined below is based on the observation that some neurocognitive results indicate (1) that a localization of specific brain regions of consciousness might be difficult, that (2) a philosophical demonstration of consciousness of individual reactions might be impossible, and (3) that consciousness can be distributed on ›true‹ and ›error‹ components of measurement so that psychometric measurement cannot unambiguously determine the latent variables that are the basis for (conscious) behavior. Thus, the parallelism implies that consciousness occurs at the borders of neurocognitive science, philosophy, and psychometrics. If consciousness is placed at the borders of the disciplines, one may ask why humans use the term ›consciousness‹ in order to provide statements on a scientifically rather intangible phenomenon. A tentative answer to this question is presented in the final part of the third, psychometric section: In a simulation study, a model whose number of parameters was larger than the number of measured variables was more robust against minor and irrelevant changes in the data. The phenomenon of a model having more parameters than measured variables is termed ›indeterminacy‹ in the psychometric literature. It was proposed that there might be an adaptive advantage for organisms to be regulated by a system that can be described as an indeterminate model that is robust against minor data changes because a robust model may enhance the continuity of behavior across time. If consciousness is an aspect of such a model, an organism equipped with consciousness may have an advantage in producing substantial continuity of behavior. Moreover, one may use the term ›consciousness‹ in order to describe a system providing behavior that is consistent across time but that can nevertheless not be unambiguously determined from available data. This is, of course, a tentative interpretation of the outlined parallelism of neurocognitive, philosophical, and psychometric perspectives. Certainly, other transdisciplinary perspectives on consciousness are possible. Nevertheless, in order to provide a basis for further research, the details of a perspective on consciousness being related to complex neuronal networks, being nearly intangible in the philosophical sense, and being psychometrically indeterminate will be presented in the following.

1. Some Neurocognitive Results on Consciousness

One strategy to cope with the complexities of research on consciousness might be to investigate manifestations and neural correlates of consciousness to manipulate conditions of conscious awareness, of conscious stimulus processing, or to use the natural variation of consciousness that occurs with sleep or brain injuries.[1] By means of this strategy investigators emphasized testable hypotheses on conscious processes. Testable hypotheses on conscious processing have regularly been investigated by means of different neurocognitive methods. For example, event-related potentials (ERP) of the electro-encephalogram have been investigated because of their high temporal resolution and functional magnetic resonance imaging (fMRI) has been per-

1 Sohn 2019.

formed because of its high spatial resolution.[2] Several experimental paradigms in the context of cognitive neuroscience and psychology have been used for the investigation of conscious stimulus processing. For example, the attentional blink paradigm refers to temporary impairment of attention occurring when two or more target stimuli have to be processed in very close temporal proximity.[3] Target stimuli are those stimuli that ask participants to react when they occur. When there is temporal proximity of two target stimuli, the second target stimulus is not always detected, which might be regarded as an ›attentional blink‹ or as a temporary impairment of conscious perception. It has been found that the accuracy in detecting two successive targets is positively correlated with intelligence.[4] The detection of two successive targets implies that the attention of participants does not ›blink‹ during the period of successive target presentation. This phenomenon has been named as (suppression of) attentional blink. The attentional blink has been addressed in ERP studies in order to investigate at which processing stage the impairment of attention occurs.[5] There are several ERP studies based on manipulations of conditions for stimulus detection that have been related to conscious awareness. Many studies investigate the amplitude of the P300 component, i.e., the size of a positive deflection occurring about 300 ms after a stimulus[6] as an indicator of conscious perception even when earlier ERPs could also mark conscious perception.[7] Although this is an example for a neurocognitive correlate of consciousness, results based on stimulus detection paradigms are typically restricted to conscious stimulus detection and do not refer to the concept of consciousness as a whole.

Although the use of experimental paradigms in cognitive neuroscience allows to measure conscious processes, the results of several experimental studies depend on the specific design and should perhaps not be generalized to consciousness as a whole. An example for the problem of specificity could be the debate on whether neural correlates of consciousness primarily activate frontal or more parietal

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2 Goense, Bohraus & Logothetis 2016.
3 Shapiro, Arnell & Raymond 1997.
4 Klein, Arend, Beauducel & Shapiro 2011.
5 E.g. Kranczioch, Debener & Engel 2003.
6 E.g. Kranczioch et al. 2003 and Lamy, Salti & Bar-Haim 2008.
7 Rutiku, Martin, Bachmann & Aru 2015.

parts of the cerebral cortex.[8] When conscious perception is measured, regions in more parietal parts of the cortex were shown to be activated. Koch et al. summarized: »… we describe recent findings showing that the anatomical neural correlates of consciousness are primarily localized to a posterior cortical hot zone that includes sensory areas, rather than to a fronto-parietal network involved in task monitoring and reporting.«[9] In contrast, when conscious motor reactions or active responses are measured, the prefrontal cortex turns out to be important. Accordingly, Odegaard et al. conclude: »The literature highlights [Prefrontal Cortex] PFC's essential role in enabling the subjective experience in perception, contra the objective capacity to perform visual tasks; conflating the two can also be a source of confusion.«[10] Thus, experiments inducing responses resulting from consciousness emphasize frontal regions and experiments inducing conscious perception are more related to activity in parietal regions of the cortex. This is not surprising as motor responses are more fronto-centrally located whereas sensory perceptions are more parietally located in the brain.[11] This demonstrates that the frontal or parietal localization of consciousness depends on the experimental paradigms. However, some neurocognitive results do not relate consciousness to specific brain areas. For example, Agnati et al. proposed mosaic networks made of hierarchically organized functional modules as a basis of consciousness. The mosaic networks allow for the realization of a wide spectrum of different neuronal assemblies, leading to the emergence of neural correlates of consciousness. Agnati et al. relate consciousness to very basic processes of neuronal activation that are widely distributed in the brain.[12] Demertzi et al. report results indicating that broad and complex neural networks can be related to consciousness: »We identified a pattern of positive and negative long-distance coordination, high modularity, with low similarity to the anatomical connectivity, potentially relevant for the support of conscious cognition (pattern 1). We also identified a pattern of low interregional dynamic coordination, low efficiency, with high similarity to anatomical connectivity, potentially specific to reduced or absent conscious

8 Koch et al. 2016 and Odegaard et al. 2017.
9 Koch et al. 2016, 307.
10 Odegaard et al. 2017, 9593.
11 Gray & McNaughton 2000.
12 Agnati et al. 2012.

processing…«.[13] These and other results on a global neuronal workspace[14] indicate that consciousness is related to a complex coordination of processes that is not necessarily limited to a specific brain region. The idea that a complex long-distance coordination might be essential for consciousness might fit with the philosophical perspectives on consciousness summarized in the next paragraph.

2. Some Philosophical Views on Consciousness

Specific neuronal regions that are related to specific conscious processes may be compatible with a philosophical mind-brain dualism in that they allow for the identification of functional-anatomic substrates underlying specific mental processes. In contrast, neurocognitive results indicating the relevance of complex distributed neuronal networks as a basis for consciousness imply that large parts of the brain are the relevant functional-anatomic substrate of consciousness. The hypothesis that large parts of the brain are the basis for consciousness is certainly not a ›bold hypothesis‹ in the sense of Popper.[15] In order to avoid trivial predictions, it might be helpful to combine the results indicating the relevance of large neuronal networks with philosophical positions that overcome the mind-brain dualism as well as neuroanatomical reductionism. According to Hagberg Wittgenstein provides a philosophical perspective on consciousness that overcomes the mind-brain dualism and the neuroanatomical reductionism.[16] It is therefore interesting to relate the neurocognitive perspective on consciousness as resulting from a complex coordination pattern with the philosophical perspective of Wittgenstein: »But isn't it our meaning it that gives sense to the sentence? (And here, of course, belongs the fact that one cannot mean a senseless series of words.) And ›meaning it‹ is something in the sphere of the mind. But it is also something private! It is the intangible something; only comparable to consciousness itself.«[17] From some neurocognitive models, consciousness is a complex pattern and from Wittgenstein's philo-

13 Demertzi et al. 2019, 3–4.
14 Dehaene, Charles, King & Marti 2014.
15 Popper 1979.
16 Hagberg 2018.
17 Wittgenstein 1958a, 358.

sophical perspective, it is an ›intangible something‹. Lehrer's more recent philosophical approach on consciousness may also fit to these ideas as he relates consciousness to the freedom of representation.[18] Lehrer assumes that conscious states provide the individual with the freedom of representation of sensory material. Accordingly, consciousness allows for a reconfiguration of sensory appearances. Lehrer wrote: »Such reconfiguration reflects our plasticity and autonomy in how we represent the external world, including the world of science. Attention to sensory detail in scientific photography enables us to distinguish artefacts of the process from features of the object photographed, for example. Exemplar representation converts input into a represented term of representation. Once the conversion takes place, we note our freedom, our autonomy, in how we represent the world and ourselves in terms of sensory materials.«[19] If consciousness is necessary for the reconfiguration of sensory representations of the world, it might also be possible that some representations are processed without consciousness, i.e., without the necessity of a reconfiguration. This can again be related to Wittgenstein: »And I want to give you the following rule of thumb: If you are puzzled about the nature of thought, belief, knowledge, and the like, substitute for the thought the expression of the thought, … This, of course, doesn't mean that we have shown that peculiar acts of consciousness do not accompany the expressions of our thoughts! Only we no longer say that they must accompany them.«[20] Thus, the perspective of consciousness as a system used in order to obtain the freedom of reconfiguration for sensory material would perfectly match with the idea that this system must not always be in action. In this sense, consciousness must not accompany the expressions of our thoughts. To sum up, Lehrer's idea that consciousness may facilitate the reconfiguration of sensory material fits to Wittgenstein's idea of consciousness as an ›intangible something‹ that can accompany the expressions of our thoughts.

Jacquette's dynamic attribution model of consciousness might also be related to this perspective.[21] Jacquette describes consciousness as follows: »Consciousness is the brain's unconscious (autonomic)

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18 Lehrer 2018.
19 Lehrer 2018, 105.
20 Wittgenstein 1958b, 42.
21 Jacquette 2018.

dynamic attribution of cognitive, including perceptual and affective data as properties to passing moments of objective mind-independent real time.«[22] Starting from this idea it may be possible to merge Lehrer's idea of consciousness that allows for a reconfiguration of sensory material with Jacquette's model of dynamic attribution of data to passing moments of time. Figure 1 represents either a cup or two faces depending on how information is integrated. If consciousness interprets the perceptual data presented in Figure 1 as a cup in one moment of time, it might be possible that consciousness interprets these data as two faces in another moment of time. Thus, the idea of time attribution may allow for the separation of moments during a constantly given configuration of sensory input. In this case attribution of time moments as proposed by Jacquette would allow to explain the reconfiguration of sensory material proposed by Lehrer by means of a second moment of sensory attribution. Moreover, a perceived change of the stimulus meaning (i.e., seeing either a cup or two faces) and knowing that one has seen two different things based on the same overall sensory material should be conceived as a result of a conscious process. So, if an individual indicates that she/he has seen two different objects in one and the same picture, one may expect that a higher form of conscious processing has occurred.

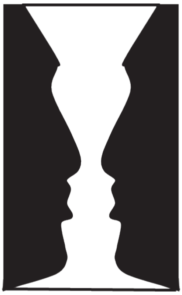

Figure 1. A cup or two faces.

Although such processes are impossible without some sort of consciousness that might be conform to some philosophical or neurocognitive model, any measurement of consciousness has to face the problem that we need a verbal indication of an individual that conscious

22 Jacquette 2018, 261.

processing has occurred. However, even explicit verbal indications may occur without a conscious understanding of the meaning of the indications. Consider, for example, Wittgenstein:

Someone says irrelevantly »That's a tree«. He might say this sentence because he remembers having heard it in a similar situation; or he was suddenly struck by the tree's beauty and the sentence was an exclamation; or he was pronouncing the sentence to himself as a grammatical example; etc., etc. And now I ask him »How did you mean that?« and he replies »It was a piece of information directed at you«. Shouldn't I be at liberty to assume that he doesn't know what he is saying, if he is insane enough to want to give me this information?[23]

Being suspicious regarding the presence of consciousness even in a rather complex verbal response, also follows from the Turing test, which is based on the rating whether a written sentence has been produced by a computer or by an individual.[24] Even when more complex versions of the Turing test have been discussed meanwhile,[25] it is sufficient in the present context to acknowledge that some sentences might be produced automatically by a computer, so that we will not expect them to be the result of human conscious processing. However, the same sentences may also be produced by a human being without knowing what he is saying, i.e., perhaps without consciousness.[26] The Turing test underlines that the occurrence of verbal sentences is no guarantee that conscious processing has occurred.

3. Representing Consciousness in Psychometric Models

3.1. Measurement Error and Consciousness

It follows from the previous paragraphs that the measurement of consciousness is quite a challenge. The reasons are: (1) Neurocognitive results indicate that consciousness is related to complex, large neuronal networks;[27] (2) verbal expressions that are typically attributed to a conscious individual may appear without conscious processing by

23 Wittgenstein 1969, 61.
24 Turing 1950.
25 Penco 2012.
26 See the above mentioned example provided by Wittgenstein 1969.
27 Demertzi et al. 2019.

the individual or may be produced by a computer,[28] (3) consciousness may be related to the attribution of perceptions to moments of time,[29] and (4) consciousness may facilitate the reconfiguration of stimulus material.[30] It should be clear from this list of arguments and results that although a large number of interesting results were obtained for specific conscious processes like perception or attention, the measurement of consciousness itself constitutes a major problem. Nevertheless, since more than one century, psychological research is devoted to the prediction of individual differences, i.e., to the identification of stable and transsituational consistent determinants of individual differences of behavior. Determinants of behavior that are stable across time and consistent across situations are often termed personality traits or individual differences of abilities.[31] It is, however, not clear how consciousness can be related to personality traits or abilities. Individual differences of consciousness are hard to conceive at the subjective level. For example, when individuals respond to questionnaires, we will typically assume that they provide perfectly conscious responses. We will usually not assume that some individuals are more conscious than others when they respond to questionnaires. It is not clear whether more careful responses are more conscious responses because individuals may consciously be careless. This shows that things are rather complex as will be illustrated when we consider the psychometric modelling of presumably conscious responses to questionnaires.

Interestingly, a substantial advancement in the measurement of traits and abilities occurred when Spearman introduced the difference between the common ability factor ›general intelligence‹ *g* and the specific abilities *s*.[32] Although the terminology was not fully developed at this time, it was already clear for Spearman that behavioral prediction will mainly be based on *g* and that *s* may be related to unpredictable aspects of behavior. Later, when factor analysis has been fully developed, the difference between common factors as determinants of behavior and unique factors comprising specific variance and

28 Wittgenstein 1958 and Turing 1950.
29 Jacquette 2018.
30 Lehrer 2018.
31 E.g. intelligence or knowledge, cf. Cattell 1987, Messick 1989 and Stern 1911.
32 Spearman 1904.

measurement error has been more clearly presented.[33] In the common factor model, the measured variables representing individual behavioral responses $\boldsymbol{x}$ are decomposed by the common factors $\boldsymbol{f}$ and the unique factors $\boldsymbol{u}$.

$$\boldsymbol{x} = \boldsymbol{\Lambda f} + \boldsymbol{\Psi u}\,. \tag{1}$$

The weights of the common factors are called ›common factor loadings‹. They are given in the loading matrix $\boldsymbol{\Lambda}$ and the weights of the unique factors are in the matrix $\boldsymbol{\Psi}$. There are some additional assumptions of the common factor model that are not discussed here.[34] It should be noted that $\boldsymbol{u}$ can be decomposed into the specific variance $\boldsymbol{s}$ and the measurement error $\boldsymbol{e}$. This yields

$$\boldsymbol{x} = \boldsymbol{\Lambda f} + \boldsymbol{\Psi}(\boldsymbol{s} + \boldsymbol{e})\,. \tag{2}$$

The idea of a measurement error that occurs with every psychological measurement and that is partly represented by the unique factors has also been emphasized in the context of classical test theory.[35] Classical test theory can be regarded as a simplification of the common factor model because only a single common factor, the so-called ›true score‹ $\boldsymbol{t}$ and the so-called ›measurement error‹ $\boldsymbol{e}$ is assumed.

$$\boldsymbol{x} = \boldsymbol{t} + \boldsymbol{e}\,. \tag{3}$$

However, the discussions on the measurement error are relevant for the common factor model as well as for classical test theory since $\boldsymbol{e}$ occurs in Equation 2 and 3. There have been several refinements and specific improvements over classical test theory that have been subsumed under the term item-response theories (Hambleton & Swaminathan, 2013). However, although mathematical and statistical refinements of measurement error are treated in a plethora of papers and books, it is hard to find a comprehensive semantic description or a conceptual, psychologically meaningful definition of measurement error.

33 Mulaik 2012.
34 Mulaik 2012.
35 Lord & Novick 1968/2008.

However, a few descriptions of the psychological meaning of measurement error are available. For example, when Lord and Novick introduced classical test theory, they started with the description of two final course examinations leading to the different results.[36] They explain that it would be a problem when students get different scores and even different orderings of scores for the same course. This leads to the assumption that there is some ›true‹ score and that the difference between the two course examinations is due to measurement error. Referring to Lazarsfeld, they consider that the trait or the ability is constant (in this sense it is regarded as ›true‹) and that some transient state of the person, resulting in differences between the examinations, is random.[37] On this basis the differences between the so-called ›true score‹ and the so-called ›error score‹ was introduced in the context of classical test theory.[38] There are, however, two possible shortcomings or misunderstandings resulting from this perspective. First, the term ›error score‹ may induce the idea that this term represents something intrinsically wrong, a real noise component. However, it has been shown that what has been termed ›error score‹ only represents the variance that is not focused in aggregation and generalization.[39] It has therefore been proposed to replace the term ›true‹ variance by ›wanted‹ or ›intended‹ variance and the term ›error‹ by ›unwanted‹ or ›non-intended‹ variance.[40] The second issue is that classical test theory and item response theories aim at describing the relationship between wanted and unwanted variance for all kinds of psychological or sociological data. They do not contain specific interpretations or methodological parameters for specific domains of measurement. The example used by Lord and Novick was from the domain of achievement and ability.[41] In their book, the common rank order of scores for the two examinations represent the wanted variance and the differences between the rank order represent the unwanted variance. It is probably quite compelling to regard the unwanted variance as a form of error variance in this specific context

36 Lord & Novick 1968.
37 Lazarsfeld 1959.
38 Lord & Novick 1968.
39 Humphreys 1962, Cronbach, Gleser, Nanda & Rajaratnam 1972 and Wittmann 1988.
40 Cattell & Radcliffe 1962 and Beauducel & Leue 2014a.
41 Lord & Novick 1968.

of achievement test. Test theories are, however, also applied for questionnaire data in the domain of personality research.[42]

As nearly all areas of psychological research implicitly or explicitly refer to some test theory, it is important to note that ›measurement error‹ or ›non-intended variance‹ can represent several aspects, which may relate to consciousness in different ways. According to Magnusson the ›measurement error‹ term may represent variance from the (1) administration of an instrument, (2) guessing, (3) scoring, and (4) lack of agreement between ›true scores‹ of different measurements.[43] The lack of agreement of true scores may be due to fluctuation of the true scores or memory effects. Other relevant aspects, not mentioned by Magnusson, are (5) social desired responding, impression management or response bias,[44] (6) states, moods, and attention/concentration during responding, and (7) previous experiences with assessment instruments. All of these aspects, with exception of (3) scoring, can be related to some conscious processing. These aspects may therefore not be random from the perspective of the individual although they are treated as if they were random in the test theoretical models. In order to provide examples for this, we will demonstrate the relationship of (4) lack of agreement between ›true scores‹ of different measurements as well as (5) response bias with the measurement of consciousness in more detail.

Lack of Agreement Between ›True Scores‹ of Different Measurements

It is worth to consider possible alterations between two measurements in the domain of questionnaire research more closely. For example, consider the sentence that might be an item of a personality questionnaire: »I can let go myself and enjoy myself at a lively party.« An individual may respond with »I strongly agree«, »I agree«, »I disagree«, and »I strongly disagree«. Let the individual respond with »I agree«. Now, the same item is presented one week later and the individual responds with »I strongly agree«. The concept of measurement error implies that there is a ›true‹ tendency to agree that lies between »I agree« and »I strongly agree« and that the measurement error explains the difference between the two measurement occasions.

42 Beauducel & Leue 2014b, Table 1.
43 Magnusson 1967.
44 Beauducel & Leue 2014a.

However, what does this imply for research on consciousness? One will usually assume a conscious answering of questionnaires although the mass of items that individuals have to respond to in several contexts may lead to careless responding (perhaps even without reading the sentence). Starting from the assumption that the sentence has been read and understood by the individual which provides a conscious response, measurement error implies that the individual was not completely inclined to respond with »I agree« at the first occasion because the individual was partly inclined to respond with »I strongly agree«. Nevertheless, the individual response was »I agree«, just in order to reach a decision. At the second occasion, the individual is again inclined to respond somewhere between »I agree« and »I strongly agree« and decided to respond with »I strongly agree«. One may figure out more complex settings with an individual that remembers the first response one week before and tries to compensate the first response by means of the second response in order to reach a true averaged response. Alternatively, the individual was sober at the first measurement occasion with some social inhibition leading to the »I agree« response and a reduced social inhibition under the influence of alcohol leading to the »I strongly agree« response at the second measurement occasion.[45] As a third idea, the individual enjoyed a nice party between the first and the second measurement occasion. Finally, the individual met someone between the measurement occasions, with whom s(he) would like to enjoy a lively party, etc. Thus, the item may represent a trait that determines that the individual tends to agree with the item and it may also represent a state to agree more or less strongly. The combined measurement of traits and states has been acknowledged and represented in Latent-State-Trait-Models.[46] It should be noted that states may change rapidly when they depend on environmental differences. For example, an individual may have low agreement with the item »I am satisfied with myself« immediately before feedback of an examination result and may have high agreement with this sentence two seconds later, when a positive feedback of the examination-result was given. Then, the difference between the responses of the individual mainly represents the feedback effect. The difference may be regarded as unwanted variance, if one is interested

45 See Lazarsfeld 1959 for a similar example.
46 Steyer, Schmitt & Eid 1999.

in an overall evaluation of self-satisfaction, but this does not imply that the variance represents an intrinsic error.

The common aspect of these examples is that the conscious response to the items might be altered. The conscious processing of the item (i.e., reading and responding) does not preclude response alterations and we expect that the item response provides us with the current conscious individual appraisal of the item content. This idea can be related to Jacquette's notion of consciousness as an attribution of data to passing moments of time. In this sense, the response alterations represented by measurement error can be related to the fidelity of conscious responses to the respective moment of time.[47] It follows from these considerations that consciousness is at least as present in the response alterations that may be represented by as the unwanted variance of unique factors as in the constant or common factor variance of the responses. Whereas psychological research on personality traits and abilities has focused on the constant part of the variance that is typically represented by the common factors of factor analysis,[48] the unique factors representing the unwanted variance or measurement error have typically been ignored. This does, however, not preclude that there are domains where especially the unique factors represents essential aspects of conscious processing. Whenever generalization of individual behavioral tendencies is not the focus and when unique responses regarding attitudes, perceptions, cognitions, or emotions need conscious processing, the unique variance or the unique factors may be of special interest for research on consciousness. Unique conscious responses could be those that are based on a specific, individual integration and interpretation of stimuli, and individual response options. To sum up, models of consciousness referring to the attribution of cognitive, perceptual, and affective data as properties to passing moments[49] as well as the perspective of consciousness as the basis for the reinterpretation of perceptions[50] underline the uniqueness of conscious experiences. Psychometric models comprising a unique term for each measurement are compatible with this perspective.

47 Jacquette 2018.
48 Mulaik 2012.
49 Jacquette 2018.
50 Lehrer 2018.

Response Bias

Presuming that items in a questionnaire do not only represent systematic variance measuring the conceptually intended construct (e.g., extraversion) but also a systematic variance that could be named as a ›bias‹ (e.g., the tendency to agree with the item content in purpose of impression management), we should aim at disentangling different types of conscious processing. According to classical test theory, we do not have a variable that corresponds to a systematic measurement component for ›bias‹ (**b**) because it is neither a ›true‹, intended variance component (**t**) nor an unsystematic error (**e**). When individuals answer items as truly as possible with regard to the instruction and with regard to memorizable situations and contexts they possibly add intentions to their answers (e.g., presenting one-self as favorable as possible). These intentions could be added consciously or unconsciously. It is part of factor analysis to disentangle **t,** as a systematic common factor, **b** as a systematic (intentional bias), and **e** as unique factor or measurement error.

3.2 Indeterminacy of Scores and Indeterminacy of Consciousness

From a formal point of view $\boldsymbol{e}$ cannot be directly determined from Equations 1–3 because there is only one measurement $\boldsymbol{x}$ but there are two numbers to be obtained: One number for the common part and one number for the unique part. Therefore, Equations 1–3 are indeterminate without further assumptions. However, Guttman introduced a definition of $\boldsymbol{t}$ for Equation 3 that has regularly been used in representations of classical test theory,[51] that is

$$E(\boldsymbol{x}) = E(\boldsymbol{t}) + E(\boldsymbol{e}) = E(\boldsymbol{t}) + 0 = \boldsymbol{t}, \tag{4}$$

where E denotes the expectation (the average value in the population). Equation 4 defines the ›true score‹ or wanted score of an individual as the expected value of the measured variables $\boldsymbol{x}$. Thus, the average of the values of an infinity of measurements $\boldsymbol{x}$ yields the true score $\boldsymbol{t}$, which is a constant in all measurements. However, in empirical settings the population of an infinity of measurements will never be

51 Guttman 1945. E.g. Lord & Novick 1968 and Zimmerman 2011.

reached, so that the average value of the available measurements will be used instead of the expectation as an estimate of the ›true score‹.[52] For p variables, this yields

$$\frac{\sum_{i=1}^{p} \boldsymbol{x}_i}{p} = \hat{\boldsymbol{t}}, \tag{5}$$

and,

$$\boldsymbol{x}_i - \hat{\boldsymbol{t}} = \boldsymbol{x}_i - \frac{\sum_{i=1}^{p} \boldsymbol{x}_i}{p} = \hat{\boldsymbol{e}}. \tag{6}$$

Although $\hat{\boldsymbol{t}}$ is an estimator of $\boldsymbol{t}$, entering Equation 3 at the left hand side of Equation 5 yields

$$\frac{\sum_{i=1}^{p} \boldsymbol{x}_i}{p} = \frac{\sum_{i=1}^{p} \boldsymbol{t}_i}{p} + \frac{\sum_{i=1}^{p} \boldsymbol{e}_i}{p} = \boldsymbol{t} + \frac{\sum_{i=1}^{p} \boldsymbol{e}_i}{p} = \hat{\boldsymbol{t}}. \tag{7}$$

As long as $\frac{\sum_{i=1}^{p} \boldsymbol{e}_i}{p} \neq 0$, the resulting $\hat{\boldsymbol{t}}$ is an indeterminate composite of $\boldsymbol{t}$ and $\boldsymbol{e}$ because any value of $\boldsymbol{t}$ can be combined with any average value of $\boldsymbol{e}_i$ in order to get a given $\hat{\boldsymbol{t}}$. The indeterminacy of classical test theory is obvious for a finite number of measurements since the error term does not vanish. However, the indeterminacy is typically not noted when classical test theory is presented for the population of measurements (in the form of Equation 4) where the error term vanishes.

In the same line, Beauducel and Leue argued that scales based on unit-weighted (e.g., personality) questionnaire items imply models that should be tested.[53] Thus, the fact that a unit-weighted sum is computed does not imply that a ›true score‹ is unequivocally determined by means of this procedure. Thus, the model implied by the unit-weighted item sum could be wrong, i.e., might not fit to the data. It is also shown in Beauducel and Leue that even unit-weighted sum scales typically imply that items are differentially important.[54] Moreover, Loevinger's critique that it is not compelling to conceive items

52 Beauducel & Leue 2014b.
53 Beauducel & Leue 2013.
54 Beauducel & Leue 2013.

(i.e., measurements) as random samples of a population of measurements has never been completely refuted.[55] From this point of view the differential weights of the measurements that have been introduced in factor analysis (Equations 1 and 2) might allow for a more realistic measurement model.

It has been shown here that the ›true score‹ estimates or wanted score estimates in the context of classical test theory are indeterminate for a finite number of measurements. For factor analysis, the indeterminacy of the common factor scores has already been noted by Wilson and has been discussed repeatedly.[56] Although the discussion has focused on the common factor scores, the unique factor scores are as indeterminate as the common factor scores.

Moreover, in conventional factor models, the indeterminate factors are considered as latent variables, as the causes of the measured variables which are considered as the effects.[57] Thus, the indeterminacy is related to modelling of a latent variable as the cause of the measured variables. Taking Equations 1–3 as models for the generation of the measurements by means of common and unique factors implies that the generation process contains information that cannot be completely reproduced by means of an analysis of the measured variables. As it is impossible to reconstruct the original scores of the generating common and unique factors from the measured variables is exactly what is implied by factor score indeterminacy. Factor score indeterminacy implies that the generation model contains more variables (factors) than the generated data set of measured variables. This complex relationship may be similar to the relationship between the observed reactions of individuals on the one hand and the internal processes, considerations, and behavioral determinants producing the observed reactions. If this complex relationship and the resulting indeterminacy is a property of conscious individuals this would imply that it would be, in principle, impossible to determine definitely whether a conscious reaction has occurred or not. It would than be easier to build up a model generating a set of measured variables for which it is impossible to reconstruct the scores of the generating variables than to reconstruct the scores of the generating variables of any

55 Loevinger 1965.
56 Wilson 1929. E.g. Guttman, 1955, Schönemann & Wang 1972 and Beauducel & Hilger 2015.
57 Bollen & Lennox 1991.

living individual. For example, it is no problem to generate a data set of artificial measured variables that are conform to the indeterminate factor model,[58] even when it is impossible to perfectly reconstruct the original generating common and unique factor scores. This would imply that the situation for research on consciousness, even when based on neurocognitive and behavioral data, is similar to the situation in the Turing test, where a number sentences are available as a basis for the attribution of (un-)consciousness to the communicator. In this sense, neurocognitive correlates of consciousness cannot definitely indicate whether consciousness was present or not. It should, however, be noted that behavioral predictions that are based on the indeterminate factor model and the indeterminate test theories are generally quite successful, as for example when intelligence scales are used for the prediction of job success.[59]

3.3. Advantages of Indeterminate Models and their Relationship to Models of Consciousness

It follows from the previous paragraph that indeterminate models contain a fundamental limitation for research because the exact scores of the original variables cannot be exactly reproduced from the number of measured variables. However, as has been noted in Section 3.1., the success of psychological behavioral predictions in several applied fields[60] are related to the use of indeterminate models since Spearman.[61] One may therefore ask whether the indeterminate factor model and the indeterminate test theories have properties that make them suitable for behavioral research. It has been noted that indeterminacy is based on the fact that the number of factors (comprising common and unique factors) is larger than the number of measured variables. It has also been noted that consciousness may operate on the basis of a system that is more complex than the resulting behavior. Therefore, possible advantages of an indeterminate psychometric model may also be advantages of a complex and, possibly indeterminate, system based on consciousness.

58 Beauducel & Hilger 2017.
59 Schmidt & Hunter 1998.
60 Schmidt & Hunter 1998.
61 Spearman 1904.

One reason why indeterminate models comprising more factors (latent variables) than measured variables could be advantageous is that common and unique factors can be equally distributed across all measured variables. Consider, for example Equations 1, where each measured variable is decomposed into at least one common factor and one unique factor. It follows that there are p unique factors when there are p measured variables (one unique factor for each variable). Since there is at least one common factor in the factor model, this implies that the sum of common and unique factors n_f is greater or equal than p +1.

But why is this an advantage? The advantage of the large number of factors is, that each measured variable, is treated equally within the model. That is, the model allows each measured variable to contain specific or unique variance as well as common variance. If a model is determinate, it cannot contain more variables than measured variables. Principal component analysis[62] is presented as an example for such a model. The aim of principal component analysis is to find components representing a maximum of the variance of the measured variables, regardless whether the variance is common or unique. In the principal component model, the number of components equals the number of measured variables, that is, $n_c = p$. Therefore, the principal component scores are determinate, i.e., they can unambiguously be computed from the measured variables. When there is at least one common component, it follows from $n_c = p$ that the number of components representing the unique variance is smaller than the number of measured variables. In consequence, it is impossible that the unique variance of each measured variable is represented by a unique component. The consequences of $n_c = p$ for the representation of the common and unique variance of the measured variables in the principal component model is demonstrated by means the following example based on artificial data.

Example: Comparison of the (Indeterminate) Factor Model and the (Determinate) PCA

A first artificial data set (Sample 1) was based on n = 2,000 cases and five normally distributed, standardized measured variables. The sample was generated by means of a random number generator of IBM

62 Harman 1967.

SPSS (Version 26). The measured variable was designed to represent 50 % of common variance and 50 % of unique variance in the population. Such a perfectly symmetric pattern will practically never occur but it is informative to see how the variances are represented when these five variables are represented in the factor model and in the principal component model. The amount of variance that the measured variables share with the unrotated common and unique factors and the unrotated principal components is represented by the squared loadings. The sample size of 2,000 cases implies that only a minimal amount of sampling error may lead to departures from squared equal population loadings of the measured variables (x1 – x5) on the common and unique factors (see Table 1).

Table 1. Squared sample loadings for the factor model and for the principal component model

	Sample 1 (n = 2,000)						Sample 2 (n = 2,000)					
Measured variables	squared factor loadings						squared factor loadings					
	common			unique			common			unique		
x1	.50	.50	.00	.00	.00	.00	.50	.50	.00	.00	.00	.00
x2	.50	.00	.50	.00	.00	.00	.50	.00	.50	.00	.00	.00
x3	.50	.00	.00	.50	.00	.00	.50	.00	.00	.50	.00	.00
x4	.50	.00	.00	.00	.50	.00	.50	.00	.00	.00	.50	.00
x5	.50	.00	.00	.00	.00	.50	.50	.00	.00	.00	.00	.50
	squared component loadings						squared component loadings					
x1	.60	.00	.03	.37	.00	-	.60	.39	.00	.00	.01	-
x2	.60	.00	.03	.04	.33	-	.60	.02	.33	.03	.01	-
x3	.60	.25	.03	.04	.08	-	.60	.05	.03	.00	.32	-
x4	.60	.00	.40	.00	.00	-	.60	.02	.00	.33	.04	-
x5	.60	.25	.03	.04	.08	-	.60	.01	.13	.13	.13	-

Note. All variances greater than zero are given in bold face.

As can be seen in Table 1, the partition of the variance (squared loadings) on the common and unique factors was as expected for the factor model in Sample 1. However, in the component model the squared loadings on the first component were considerably larger than the squared loadings on the common factor. Although this has been regarded as an indication that the principal component model overestimates the loadings,[63] the focus of the present demonstration is on the difference between the unique variance which is equally distributed across the measured variables in the factor model whereas the distribution of the variance on the components 2–5 is rather unequal in the component model.

Sample 2 was generated in order to investigate the robustness of the variances explained by the factor model and the principal component model. Therefore, Sample 2 was based on the same population model. Moreover, Sample 2 was identical to Sample 1 with the only exception that the value of the first case in the first value was increased by 0.01 (it was -0.03 in Sample 1 and -0.02 in Sample 2). No other values were changed so that 99.99 % of the values in Sample 2 were identical to the values in Sample 1. As expected, the variance distribution on the common factor and the unique factor was not altered by this minimal modification. The variance explained by the first component also remained unchanged. However, the variance explained by the components 2–5 was quite different. Since the modification of the sample was minimal, this indicates that the component model does not result in robust estimations of the more specific variances.

The variability of parameters of the indeterminate factor model and of the determinate principal component model can also been shown in a more complex simulation study. First, a population of three common factors was generated that can also be described by three salient principal components. The corresponding population loadings are given in the Appendix (Table A1). From this population 1,000 samples with n = 400 cases were drawn and submitted to common factor analysis as well as to principal component analysis. For each sample, a factor analysis and a principal component analysis was performed and -as in the previous example with two samples- the variability (standard deviation) of the unique factor loadings and the standard deviation of the non-salient principal component loadings was investigated. The resulting standard deviation of the unique factor loadings ($\mathbf{\Psi}$) was s =.033 whereas the standard deviation of the non-salient principal component loadings ($\mathbf{N}$) was s =.200. For the common factor loadings, the standard deviation was s =.301 whereas it was s =.364 for the salient principal component loadings. Thus, the standard deviations of the factor loadings were smaller than the stan-

63 Snook & Gorsuch 1989.

dard deviations for the principal component loadings. This indicates that the principal component loadings depend more on sampling error than the factor loadings. As sampling error represents a variability that is due to noise, a model that is less affected by sampling error can be regarded as statistically more robust. Although more complex simulation studies would be necessary in order to explore the conditions of the instability of the component variances/loadings, it is demonstrated here that the indeterminate factor model may yield more robust model parameters than the determinate component model.

The loadings of the common factors have been compared with the corresponding principal component loadings in more comprehensive studies on the effect of changing contexts of variables on results. These studies have found that the common factor loadings were more robust against changes of the context of variables than the respective component loadings.[64] Although these studies did not investigate the robustness of the unique loadings and their corresponding principal components, they also found that the indeterminate factor model was more robust than the determinate principal component model.

The initial question of this section was, whether there could be possible advantages of indeterminate models. In light of the example shown here and in light of previous research one may conclude that the robustness of model parameters may be enhanced for indeterminate models. It could be that the larger number of model parameters allows to represent small empirical variations more conveniently, which may explain the enhanced robustness of indeterminate models. This may—in turn—explain the success of such models in the prediction of human behavior.

4. Conclusion

Since indeterminate models are robust across minor parameter variations, it is possible to assume that consciousness—which may help to provide behavior that is consistent across time—can be described by models having more parameters than can be measured by means of observed variables. Such models of consciousness are indeterminate in that the original scores representing consciousness cannot be

64 Widaman 1993 and Beauducel 2000.

completely reproduced from the measured data. As mentioned in the introduction, one may speculate whether it is adaptive for organisms to be based on more parameters than necessary for the generation of immediate behavioral outcome. Being robust against minor changes of the observed variables may facilitate temporal continuity of behavior. If consciousness is related to the fact that the number of model parameters is larger than the number of observed outcome variables, consciousness may be related to behavioral continuity. The perspective outlined in this chapter implies that neurocognitive correlates of consciousness cannot definitely indicate whether consciousness was present or not. Moreover, this perspective is compatible with the results of the previous sections, namely (Section 1.) that consciousness is related to a complex coordination of processes that is not necessarily limited to a specific brain region, that (Section 2.) consciousness is related to very basic processes like the dynamic attribution of data to passing moments of time and to the reconfiguration of stimulus material while the consciousness of an individual reaction cannot unambiguously be assumed, and (Section 3.) that conscious reactions can be the basis of variance components representing the true part of the variance as well as of variance components representing measurement error. This distribution of consciousness on several variance components implies that the indeterminate factor model may be appropriate for the description of conscious behavior. It remains to be explored whether the ideas presented here can be related to aspects of indeterminacy that are discussed in other fields,[65] especially in the field of artificial intelligence.[66]

References

Agnati, Luigi Francesco, Guidolin, Diego, Cortelli, Pietro, Genedani, Susanna, Cela-Conde, Camilo, & Fuxe, Kjell (2012): Neuronal correlates to consciousness. The »Hall of Mirrors« metaphor describing consciousness as an epiphenomenon of multiple dynamic mosaics of cortical functional modules, in *Brain Research* 1476, 3–21. https://doi.org/10.1016/j.brainres.2012.01.003

Beauducel, André (2001): On the generalizability of factors: The influence of changing contexts of variables on different methods of factor extraction, in *Methods of Psychological Research* 6, 1–24.

[65] del Val 2020.
[66] Wit et al. 2018.

Beauducel, André & Hilger, Norbert (2015): Extending the debate between Spearman and Wilson 1929: When do single variables optimally reproduce the common part of the observed covariances?, in *Multivariate Behavioral Research* 50, 555–567.

Beauducel, André & Hilger, Norbert (2017): On the bias of factor score determinacy coefficients based on different estimation methods of the exploratory factor model, in *Communications in Statistics – Simulation and Computation* 46, 6144–6154. https://doi.org/10.1080/03610918.2016.1197247

Beauducel, André & Leue, Anja (2013): Unit-weighted scales imply models that should be tested!, in *Practical Assessment, Research & Evaluation* 18(1).

Beauducel, André & Leue, Anja (2014a): *Grundlagen der Psychologischen Diagnostik*, Göttingen: Hogrefe.

Beauducel, André & Leue, Anja (2014b): Testing the assumption of uncorrelated errors for short scales by means of structural equation modeling, in *Journal of Individual Differences* 35, 201–211.

Bollen, Kenneth & Lennox, Richard (1991): Conventional wisdom and measurement: A structural equation perspective, in *Psychological Bulletin* 110, 305–314. https://doi.org/10.1037/0033-2909.110.2.305

Casali, Adenauer G., Gosseries, Olivia, Rosanova, Mario, Boly, Mélanie, Saasso, Simone, Casali, Karina R., Casarotto, Silvia, Bruno, Marie-Aurélie., Laureys, Steven, Tononi, Giulio, & Massimini, Marcello (2013): A theoretically based index of consciousness independent of sensory processing and behavior, in *Science Translational Medicine* 5, 198: 198ra105.

Cattell, Raymond Bernard & Radcliffe, John A. (1962): Reliabilities and validities of simple and extended weighted and buffered unifactor scales, in *The British Journal of Statistical Psychology* 15, 113–128.

Cronbach, Lee J., Gleser, Goldine C., Nanda, Harinder & Rajaratnam, Nageswari (1972): *The Dependability of Behavioral Measurements: Theory of Generalizability of Scores and Profiles,* New York: Wiley.

Dehaene, Stanislas, Charles, Lucie, King, Jean-Rémi, & Marti, Sébastien (2014): Toward a computational theory of conscious processing, in *Current Opinion in Neurobiology* 25, 76–84.

Del Val, Jaime (2020): Beyond error: Philosophy of indeterminacy in the age of algorithms, in Popat, Sita and Whatley, Sarah (eds.): *Error, ambiguity, and creativity: A multidisciplinary reader*. Cham, Switzerland: Springer Nature, 85–106. https://doi.org/10.1007/978-3-030-39755-5

Demertzi, Athena, Tagliazucchi, Enzo, Dehaene, Stanislas, Deco, Gustavo, Barttfeld, Pablo, Raimondo, Federico, Martial, Charlotte, Fernández-Espejo, Davinia, Rohaut, Benjamin, Voss, Henning U., Schiff, Nicholas D., Owen, Adrian M., Laureys, Steven, Naccache, L., Sitt, Jacobo Diego (2019): Human consciousness is supported by dynamic complex patterns of brain signal coordination, in *Science Advances* 5: eaat7603.

Goense, Jozien, Bohraus, Yvette, & Logothetis, Nikos K. (2016): fMRI at high spatial resolution: Implications for BOLD-Models, in *Frontiers in Computational Neuroscience* 10, 66. https://doi.org/10.3389/fncom.2016.00066

Gray, Jeffrey A., & McNaughton, Neil (2000): *The Neuropsychology of Anxiety*, Oxford: University Press.
Guttman, Louis (1945): A basis for analyzing test-retest reliability, in *Psychometrika* 10, 255–282.
Guttman, Louis (1955): The determinacy of factor score matrices with applications for five other problems of common factor theory, in *British Journal of Statistical Psychology* 8, 65–82. https://doi.org/10.1111/j.2044-8317.1955.tb00321.x
Hagberg, Garry L. (2018): Wittgenstein and the concept of consciousness, in Jacquette, D. (ed.), *The Bloomsbury Companion to the Philosophy of Consciousness*, Bloomsbury Publishing. Kindle-Version, 57–77.
Hambleton, Ronald K. & Swaminathan, Hariharan (2013): *Item Response Theory: Principles and Applications*, New York: Springer.
Harman, Harry H. (1976): *Modern Factor Analysis* (3rd ed.), Chicago, : The University of Chicago Press.
Humphreys, Lloyd G. (1962): The organization of human abilities, in *American Psychologist* 17, 475–483.
Jacquette, Dale (2018): Cracking the Hard Problem of Consciousness. In Jacquette, D. (ed.), *The Bloomsbury Companion to the Philosophy of Consciousness*, Bloomsbury Publishing. Kindle-Version, 258–287.
Klein, Christoph, Arend, Isabel C., Beauducel, André, & Shapiro, Kimron L. (2011): Individuals differ in the attentional blink: Mental speed and intra-subject stability matter, in *Intelligence* 39, 27–35.
Koch, Christof., Massimini, Marcello, Boly, Melanie, & Tononi, Giulio (2016): Neural correlates of consciousness: Progress and problems, in *Nature Reviews Neuroscience* 17, 307–321.
Kranczioch, Cornelia, Debener, Stefan, & Engel, Andreas K. (2003): Event-related potential correlates of the attentional blink phenomenon, in *Cognitive Brain Research* 17, 177–187.
Lamy, Dominique, Salti, Moti, Bar-Haim, Yair (2009): Neural correlates of subjective awareness and unconscious processing: an ERP study, in *Journal of Cognitive Neuroscience* 21, 1435–1446.
Lazarsfeld, Paul Felix (1959): Latent structure analysis, in Koch, S. (ed.), *Psychology: a study of a science* Vol. 3. New York: McGraw-Hill.
Lehrer, Keith (2018): Conscious exemplars as freedom of representation, in Jacquette, D. (ed.), *The Bloomsbury Companion to the Philosophy of Consciousness*, Bloomsbury Publishing. Kindle-Version, 104–122.
Loevinger, Jane (1965): Person and population as psychometric concepts, in *Psychological Review* 72, 143–155.
Lord, Frederic M. & Novick, Melvin R. (1968/2008): *Statistical Theories of Mental Test Scores*, Addison-Wesley, Menlo Park.
Magnusson, David (1967): *Test Theory* (2nd ed), London: Addison-Wesley.
Mulaik, Stanley A. (2012): *Foundations of Factor Analysis*, New York: Taylor & Francis.

Odegaard, Brian, Knight, Robert Thomas, & Lau, Hakwan (2017): Should a few null findings falsify prefrontal theories of conscious perception?, in *The Journal of Neuroscience* 37 (40), 9593–9602.
Penco, Carlo (2012): Updating the Turing test. Wittgenstein, Turing and symbol manipulation, in *Open Journal of Philosophy* 2, 189–194.
Popper, Karl Raimund (1979): *Objective Knowledge. An Evolutionary Approach* (revised ed.), Oxford: Oxford University Press.
Rutiku, Renate, Martin, M., Bachmann, Talis, & Aru, Jaan (2015): Does the P300 reflect conscious perception or its consequences?, in *Neuroscience* 298, 180–189.
Schönemann, Peter H., & Wang, Ming-Mei (1972): Some new results on factor indeterminacy, in *Psychometrika* 37, 61–91. https://doi.org/10.1007/BF02291413
Schmidt, Frank L. & Hunter, John E. (1998): The validity and utility of selection methods in personnel psychology. Practical and theoretical implications of 85 years of research findings, in *Psychological Bulletin* 124, 262–274.
Shapiro, Kimron L., Arnell, Karen A., & Raymond, Jane E. (1997): The attentional blink: A view on attention and a glimpse on consciousness, in *Trends in Cognitive Science* 1, 29–296.
Snook, Steven C. & Gorsuch, Richard L. (1989): Component analysis versus common factor analysis: A Monte Carlo study, in *Psychological Bulletin* 106, 148–154.
Sohn, Emily (2019): Decoding Consciousness, in *Nature* 571, S1-S5. https://doi.org/10.1038/d41586-019-02207-1
Spearman, Charles (1904): »General intelligence.« Objectively determined and measured, in *The American Journal of Psychology* 15, 201–292.
Steyer, Rolf, Schmitt, Manfred & Eid, Michael (1999): Latent state-trait theory and research in personality and individual differences, in *European Journal of Personality* 13, 389–408.
Turing, Alan Mathison (1950): Computing Machinery and Intelligence, in *Mind* 236, 430–460.
Widaman, Keith F. (1993): Common factor analysis versus principal component analysis: Differential bias in representing model parameters?, in *Multivariate Behavioral Research* 28, 263–311.
Wilson, Erik B. (1929): Comment on Professor Spearman's note, in *Journal of Educational Psychology* 20, 217–223. https://doi.org/10.1037/h0071925
Wit, Andrew John, Vasey, Lauren, Parlac, Vera, Marcu, Mara, Jabi, Wassim, Gerber, David, Daas, Mahesh, & Clayton, Mark (2018): Artificial intelligence and robotics in architecture: Autonomy, agency, and indeterminacy, in *International Journal of Architectural Computing* 16, 245–247. https://doi.org/10.1177/1478077118807266
Wittgenstein, Ludwig (1958a): *Philosophical Investigations*, Oxford, UK: Blackwell.
Wittgenstein, Ludwig (1958b): *The Blue and Brown Books: Preliminary Studies for the "Philosophical Investigations"*, Oxford, UK: Blackwell.
Wittgenstein, Ludwig (1969): *On Certainty*, Oxford, UK: Blackwell.

Wittmann, Werner W. (1988): Multivariate reliability theory: Principles of symmetry and successful validation strategies, in Nesselroade, J. R. & Cattell, R. B. (eds.): *Handbook of Multivariate Experimental Psychology* 2nd ed. New York: Plenum, 505–560.
Zimmerman, Donald W. (2011): Sampling variability and axioms of classical test theory, in *Journal of Educations and Behavioral Statistics* 36, 586–615. https://doi.org/10.3102/1076998610397052

Appendix

Table A1. Population common factor loadings and population principal component loadings

	Common factor loadings			Salient component loadings		
	F1	F2	F3	C1	C2	C3
x1	.55	.00	.00	.66	.00	.00
x2	.55	.00	.00	.66	.00	.00
x3	.55	.00	.00	.66	.00	.00
x4	.55	.00	.00	.66	.00	.00
x5	.55	.00	.00	.66	.00	.00
x6	.55	.00	.00	.66	.00	.00
x7	.00	.53	.00	.00	.65	.00
x8	.00	.53	.00	.00	.65	.00
x9	.00	.53	.00	.00	.65	.00
x10	.00	.53	.00	.00	.65	.00
x11	.00	.53	.00	.00	.65	.00
x12	.00	.53	.00	.00	.65	.00
x13	.00	.00	.50	.00	.00	.63
x14	.00	.00	.50	.00	.00	.63
x15	.00	.00	.50	.00	.00	.63
x16	.00	.00	.50	.00	.00	.63
x17	.00	.00	.50	.00	.00	.63
x18	.00	.00	.50	.00	.00	.63

Note. Salient loadings are given in bold face.