CHAPTER 8

Applying GTSP-algorithms in maritime patrolling missions that require mutual support

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Abstract
The goal of a maritime patrol is to detect, locate and identify (opposing) vessels. In some of these missions, the patrol aircraft may need the support of another aircraft to counter a possible threat. This support may consist of using capabilities of the other aircraft, such as its sensors, communications or its weapons. Our work aims to optimise maritime patrol routes for two or more drones, while providing the possibility of mutual support. We show that this routing problem including mutual support can be modelled as a generalised travelling salesman problem (GTSP). We investigate the costs of requiring mutual support and compare it to the costs of using separate drones that detect and identify vessels in the area of operations.

Keywords: Generalised travelling salesman problem, Maritime patrolling, Mutual support, Unmanned aerial vehicles

8.1. Introduction

In this chapter, we consider the problem of routing two drones, while guaranteeing the possibility for mutual support at any point in time. We therefore first consider maritime patrols and discuss and illustrate the need for mutual support in modern warfare. We then discuss related routing problems that may be found in the literature, which are variants of the classical travelling salesman problem (TSP).

8.1.1. Maritime patrolling and mutual support

The Exclusive Economic Zone (EEZ) is an area beyond and adjacent to a coast or territorial sea to a limit of 200 nautical miles from the baseline. In this zone, the coastal state can practice sovereign rights over the exploration, conservation and management of resources and other economic activities, such as the production of tidal or wind energy. The coastal state is mainly responsible for the preservation of living resources in the EEZ. It has judicial and supervisory powers within its EEZ to counter dumping of waste from ships and pollution from deep-sea activity.[1] The EEZ zone was first formalised in international law. It was a compromise between maritime powers and coastal states. As part of the United Nations Convention on the Law of the Sea (UNCLOS) "deal package" it recognised coastal states' rights to economic resources within the EEZ, while still protecting (to some extent) access by other states to the area for non-economic purposes.[2] Nevertheless, military activities in the EEZ have been controversial as the UNCLOS text does not explicitly address military activities, leaving room for different interpretations. This has resulted in several diplomatic and military conflicts between some of the world's major powers and has been the subject of much academic debate. The focal point of disagreement is the level to which UNCLOS allows other states to carry out military activities, including surveillance, within a coastal state's EEZ. The concerns are high: while some see military activities as an unacceptable threat to a coastal state's sovereignty, others see them as securing and protecting maritime trade and undersea cables, and of crucial importance to the global economy and global security.[3]

Maritime patrol is the action of employing aerospace means to detect, locate, identify, monitor, or neutralise opposing vessels. These vessels can operate in open sea, inland waters or other maritime spaces of interest to naval operations. Maritime patrol comprises several activities that require integrated and synchronised cooperation with friendly naval forces. It may also include the supervision of jurisdictional activities carried out by the Navy.[4] An aerial maritime surveillance (or maritime patrol) mission can be divided into five tasks: detection (locating something of interest), classification (boat, iceberg, oil slick, etc.), identification (name and nationality of a vessel), inspection (determination of vessel or object activity), and enforcement (issue warning or evidence collection).[5] There are many classes of maritime patrol missions: patrol and law enforcement, investigation and evidence collection, maritime search and rescue, monitoring of oil spilling and pollution of ship discharge, emergency action, patrolling and examination of the buoy, survey of the channel, and international maritime supervision or maritime patrol.[6] All these activities can be carried out by active (radar) and passive (thermal and electro-optical) sensors airborne on the aircraft and are sent to a control centre by a datalink system.

Several studies in the literature have shown the substitution of manned vehicles for unmanned vehicles in various military missions, including maritime patrol missions.[7,8,9] This replacement has been taking place due to several advantages such as: greater autonomy, performance (e.g., agility), overload durability, and stealth capacity.[10] Other advantages are the possibility to admit aircraft losses from a tactical formation in favour of the victory of a squadron in a given combat scenario[11], and improved accuracy when firing a missile.[12] However, there are a few practical downsides of this substitution. For example, drones, also known as unmanned aerial vehicles, may be more susceptible to electronic interference (for example jamming, which can reduce radar range, disrupt communication/data link, interfere with the aircraft control, and interfere with the drone's passive sensors) or mechanical interference (use of weapons). In a manned aircraft, electronic interference would be easily recognised by an experienced operator. An attack with kinetic weapons would be unlikely to occur on a manned aircraft, due to the legal implications. This illustrates that drones are more susceptible to interference.

In order to try to guarantee the execution of an effective and robust maritime patrol, which aims to collect data necessary for the control and surveillance of the maritime area, an additional drone can be used to guarantee mutual support between the two aircraft. Mutual support is a commitment within a flight of two or more aircraft that supports the mission objectives of the flight. In modern combat, mutual support is directly related to situational awareness.[13,14] But, in addition, an effective mutual support arrangement will also allow a maritime patrol to generate offensive power to neutralise hostile intentions, thereby increasing the survivability of the team of aircraft. The mutual support may also provide an operational capability in the event of the inoperability of one of them. That is, in the event of a failure of electronic equipment in one of the drones, the other drone must be at such a distance that ensures it can continue to perform the task that the failing drone cannot perform. In this paper we assume that the drones constantly maintain a maximum separation distance, while carrying out their patrol independently. This model assumption, the maximum separation distance, aggregates all the various aspects and types of mutual support like protection and sustaining operational capabilities. An example of this capability is the SAR (synthetic-aperture radar) whose functions are identification and surveillance, data link for command and control, and exchange of operational messages.

To summarise, the objective of this work is to optimise maritime patrol routes for two or more drones, providing the possibility of mutual support. We assume that the drones have been assigned a number of contacts in the area of operation which they have to investigate. We show that this routing problem including mutual support can be modelled as a generalised travelling salesman problem (GTSP). We prove that it can be turned into a regular TSP. We also investigate the costs of requiring mutual support and compare it with the costs of using separate drones that detect and identify vessels in the area of operations.

8.1.2. Related literature

Most of the works related to the routing of maritime patrols that can be found in the literature aim to optimise the flight time, the fuel consumption, or aim at maximising the amount of collected data. In the case of a swarm of drones, it seeks to optimise detection and tracking of targets.[15] A Lagrangian swarm model that has the capability of covering large areas of the sea effectively is presented by Kumar and Vunualailai.[16] The controllers derived in this work generate a linear formation which, if applied to dynamical systems, will have the ability to be a very good model for the surveillance of an Economic Exclusive Zone, as well as for search and rescue. The monitoring of trawler activities in Kuala Keda was studied by Suteris et al.[17] They create a route optimisation method for unmanned air vehicles for maritime surveillance. The aim is to find a short tour, in terms of time, to cover all locations at sea either by boat or by both a boat and a drone. Optimisation of a drone's trajectory for maritime radar wide area surveillance is discussed by Brown and Anderson.[18] The considerations of the dynamics, propulsion, and mission requirements of a fixed-wing drone, as well as a maritime surveillance radar, provide a method to obtain the fuel consumption, probability of detection, and revisit time for a given trajectory.

The main contribution of our work concerns the optimisation of maritime patrol routes while maintaining a maximum distance between two drones. This distance is based on the range of their sensors. This way, the two drones can provide mutual support throughout the route, also in a hostile environment. Comparable models are studied by Murray and Chu for example.[19] Their one considers the flying sidekick travelling salesman problem for the optimisation of drone-assisted parcel delivery. Carlsson and Song investigated routing problems where a drone provides service to customers while making return trips to a truck that is itself moving.[20] In other words, a drone picks up a package from the truck (which continues on its route), and after delivering the package, the drone returns to the truck to pick up the next package. Similarly, Agatz et al. study last-mile delivery concepts in which a truck collaborates with a drone.[21]

8.1.3. Travelling salesman problems

In this paper, we use several versions of the travelling salesman problem (TSP). In TSP, we are given a set of vertices V, with $|V| = n$, and costs $c(i,j)$ for vertices $i,j \in V$. The goal is to find a tour, that visits every vertex at least once, of minimum total cost. We discuss two variants: 2-person TSP (2TSP), and generalised TSP (GTSP).

In an instance of 2-person TSP, we additionally have a root vertex.[22] Now, a feasible solution consists of two tours, starting and ending in the root, such that every vertex is visited at least once. The goal is to minimise the maximum cost of the two tours. We will use this variant when we consider the inefficiency caused by demanding mutual support.

In the generalised TSP[23], we are given a set of node sets $S_1, \ldots, S_m \subseteq V$. A feasible solution is a tour, that visits at least one vertex from each node set. Again, the goal is to minimise the tour cost. We will transform our routing problem, including mutual support, into GTSP. Then, we show that any instance of GTSP can be reduced to an instance of TSP.

8.1.4. Outline

Now that we have illustrated the need for mutual support during maritime patrolling, we provide a mathematical model in Section 8.2. There we define the mathematical problem of routing two drones, while demanding mutual support at any point in time. Our model can be described as a generalised travelling salesman problem (GTSP). In Section 8.3, we will show how to reduce any instance of GTSP into an instance of a regular, albeit asymmetric TSP. Mutual support comes at a cost compared to a patrol of two independently routed drones. Therefore, we study the inefficiency of demanding mutual support, both theoretically and based on real-life instances. In Section 8.5, we discuss how to generalise our approach to more than two drones, and what issues should be considered when doing this. Finally, we give our conclusion and present operational considerations.

8.2. Problem description and reduction to GTSP

We call our problem the 2-travelling salesman problem with mutual support, abbreviated to 2TSP-MS. We assume that the drones are identical, and that we will operate at a constant altitude. Hence, in an instance of 2TSP-MS, we are given a set of n points $\mathrm{P} \subset \mathbb{R}^2$, and a root $r \in \mathbb{R}^2$. Let $d(x,y)$ denote the Euclidean distance between points $x \in P$ and $y \in P$. Each of the two drones flies a tour on a subset of the points, where it can vary its speed over time, with a maximum speed of S. The distance between the two drones has to be at most R at any point in time. Each point has to be visited at least once. The goal is to find two tours and corresponding speeds such that the time spent is minimised.

To keep the solution space comprehensible, we assume that the drones have to visit points from P simultaneously. This mimics the assumptions in the related literature, where drones return to the truck at delivery points only.[24,25,26] Of course, one may construct examples that this assumption comes at some extra costs. By matching the speeds of the two drones, we can guarantee that they will always be within a distance of R units apart, without explicitly tracking the drones over time (see Theorem 1). We also allow a drone to wait at a point. This implies that the drone will loiter around a location if it has to fly at some minimum speed. To accommodate the need to visit points that are at distance more than R from any other point in P, we assume that the two drones can visit the same point simultaneously. Finally, we assume that there is no limit on the endurance or range of our drones. Note that this final assumption might restrict the speed at which we can fly. This implies that we might choose a lower maximum speed S. Moreover, the maximum speed also depends on the altitude we have chosen.

An instance and a feasible solution of 2TSP-MS is illustrated in Figure 8.1. In the solution, drone 1 flies the route A, B, D, F, and drone 2 flies the route C, E, F. First, drone 1 flies at maximum speed S from r to A and B. Drone 2 flies at a lower speed such that the drones arrive at points A and C simultaneously. Then, drone 2 waits at C until drone 1 has reached point B. Both then continue at maximum speed to their next destinations D and E. From D to F, drone 1 again flies at maximum speed, while drone 2 lowers its speed from E to F to arrive at the same time at point F. Finally, both drones fly at maximum speed back to the root.

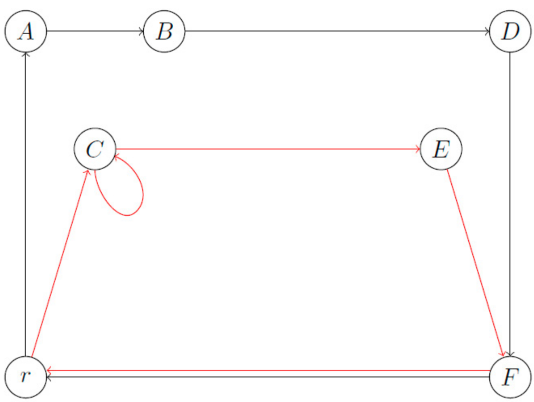

Fig. 8.1. An instance and feasible solution of 2TSP-MS.

To transform an instance of 2TSP-MS into an instance of GTSP, we use the notion of a configuration. If we use configuration (x, y), it means that drone 1 visits $x \in P$, and drone 2 visits $y \in P$ simultaneously. We only consider configurations (x, y) that satisfy $d(x, y) \leq R$. If the drones move from configuration (A_1, B_1) to configuration (A_2, B_2), both drones will travel in a straight line. The drone that has to travel the largest distance flies at maximum speed S. To make sure that the two drones arrive at their destinations A_2 and B_2 at the same time, the other drone then can adjust its speed to, for example, the speed S' that is specified in the proof of Theorem 1. This also ensures that at any point in between, the two drones are at most distance R from each other.

Theorem 1
If the two drones move from configuration (A_1, B_1) to configuration (A_2, B_2) as described above, they spend time

$$\frac{\max\left\{d(A_1, A_2), d(B_1, B_2)\right\}}{S},$$

while ensuring that the distance between the drones is at most R at any point in time.
Proof: We first show that moving from configuration (A_1, B_1) to configuration (A_2, B_2) takes $\max\left\{d(A_1, A_2), d(B_1, B_2)\right\}/S$ time. The drone that has to travel the largest distance travels at maximum speed S. It therefore arrives at time $\max\left\{d(A_1, A_2), d(B_1, B_2)\right\}/S$. The other drone can adjust its speed to

$$S' = \frac{\min\left\{d(A_1, A_2), d(B_1, B_2)\right\}}{\max\left\{d(A_1, A_2), d(B_1, B_2)\right\}} S < S$$

Moving at this speed, it will also arrive at its destination at time $\max\left\{d(A_1, A_2), d(B_1, B_2)\right\}/S$.
To show that the distance between the two drones is at most R at any point in time, let the vectors $\vec{a}_1, \vec{a}_2, \vec{b}_1, \vec{b}_2 \in \mathbb{R}^2$ denote the coordinates of points A_1, A_2, B_1, and B_2, respectively. For $0 \leq t \leq \max\left\{d(A_1, A_2), d(B_1, B_2)\right\}/S$, the position of drone 1 can be parametrised as

$$\left(1 - \frac{t}{\max\left\{d(A_1, A_2), d(B_1, B_2)\right\}/S}\right)\vec{a}_1 + \frac{t}{\max\left\{d(A_1, A_2), d(B_1, B_2)\right\}/S}\vec{a}_2,$$

while the position of drone 2 can be parametrised as

$$\left(1 - \frac{t}{\max\left\{d(A_1, A_2), d(B_1, B_2)\right\}/S}\right)\vec{b}_1 + \frac{t}{\max\left\{d(A_1, A_2), d(B_1, B_2)\right\}/S}\vec{b}_2.$$

Note that at $t = 0$, drone 1 is at point A_1 and drone 2 is at point B_1. Similarly, at $t = \max\left\{d(A_1, A_2), d(B_1, B_2)\right\}/S$, drone 1 is at point A_2 and drone 2 is at point B_2. Moreover, both drones move at a constant speed. The distance between the drones at any time $0 \leq t \leq \max\left\{d(A_1, A_2), d(B_1, B_2)\right\}/S$ is at most R, as follows from the following reasoning, where we let $u = t/\left(\max\left\{d(A_1, A_2), d(B_1, B_2)\right\}/S\right)$.

$$\begin{aligned}
\left\|(1-u)\vec{a}_1 + u\vec{a}_2 - \left((1-u)\vec{b}_1 + u\vec{b}_2\right)\right\| &= \left\|(1-u)\left(\vec{a}_1 - \vec{b}_1\right) + u\left(\vec{a}_2 - \vec{b}_2\right)\right\| \\
&\leq \left\|(1-u)\left(\vec{a}_1 - \vec{b}_1\right)\right\| + \left\|u\left(\vec{a}_2 - \vec{b}_2\right)\right\| \\
&= \left(1-u\right)\left\|\left(\vec{a}_1 - \vec{b}_1\right)\right\| + u\left\|\left(\vec{a}_2 - \vec{b}_2\right)\right\| \\
&\leq (1-u)R + uR \\
&= R
\end{aligned}$$

∎

Using Theorem 1, a feasible solution to 2TSP-MS is a sequence of configurations (A_1, B_1), $(A_2, B_2), \ldots, (A_k, B_k)$ such that for each point $x \in P$ we have $x \in \{A_i, B_i\}$ for at least one i. In this case, we say that the configuration contains x. The sequence represents the route the two drones are flying, in which they start and end at the root r. For all $x \in P$, we allow (x, x) to be a configuration: both drones simultaneously visit x. This, for example, enables us to visit an isolated point $x \in P$ (no y exists such that $d(x, y) \leq R$). It also may occur that one of the drones keeps hovering at the same point y, while the other drone flies from point z to another point q. Therefore (y, z) and (y, q) can be two consecutive configurations in such a sequence.

We now shall formulate 2TSP-MS as a generalised travelling salesman problem (GTSP), using the following construction.

- Create a vertex (x, y) for each configuration (x, y) with $d(x, y) \leq R$. In that case, also (y, x) is a vertex.
- Create a directed arc between any pair of vertices (x, y) and (u, v).
- Define the costs of an arc from (x, y) to (u, v) as $\max\{d(x, u), d(y, v)\}/S$. The costs of the arc between the root r and (x, y) is equal to $\max\{d(r, x), d(r, y)\}/S$.
- Define for each of the n points x a nodeset S_x that contains all vertices that contain x.

This results in a complete (see Theorem 1), bi-directed graph, with overlapping nodesets. Feasible solutions for this GTSP have to take at least one vertex from each nodeset, starting and ending at r. It is easy to verify that a feasible solution to our original problem is a feasible solution of the GTSP, and vice versa. So, 2TSP-MS is equivalent to GTSP on the created instance. The reduction is illustrated in Figure 8.2.

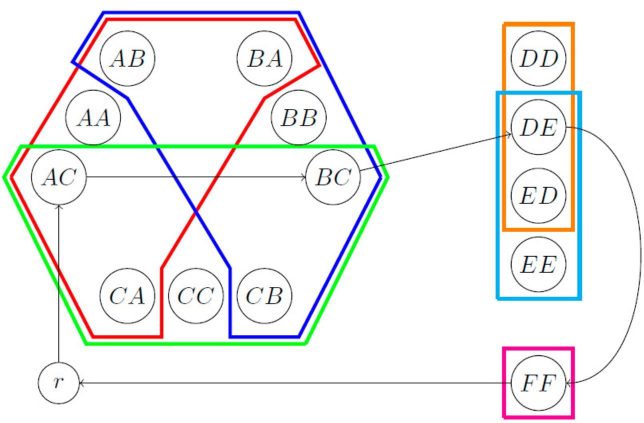

Fig. 8.2. Illustration of reduction from 2TSP-MS to GTSP, where vertex XY corresponds to configuration (x, y). The nodesets are illustrated by areas with colored borders. The black arcs depict a solution of the GTSP-instance, that corresponds to the solution of 2TSP-MS in Figure 8.1.

Although the focus of this paper is not on computational issues, we refer to research that investigates efficiency of meta-heuristics for the GTSP. Ben Nejma and Mhalla consider local search algorithms to solve the GTSP.[27] Cosma et al. present a novel Hybrid Genetic Algorithm (HGA) for solving the GTSP.[28] Finally, Wu et al. use genetic algorithms that can solve the GTSP directly without the need of intermediate transformations to TSP, the issue of the next section.[29]

8.3. Reduction to TSP

To solve our problem with two drones that require mutual support, we could transform the GTSP of the previous section into a travelling salesman problem. We provide the transformation in two steps:

- Transform the GTSP into a GTSP without overlapping nodesets.
- Apply the reduction from Noon and Bean from GTSP without overlapping nodesets to TSP.[30]

Remember that in the GTSP of the previous section, vertices (x,y), with $x,y \in P$ and $d(x,y) \leq R$ are grouped into nodesets. Our nodesets overlap. For example, S_x, as well as nodeset S_y, contains the vertex (x,y). To eliminate the overlap between two nodesets, we use the following construction.

- Make two copies of each vertex (x,y) with $x \neq y$: $(x,y)^x$ and $(x,y)^y$.
- Introduce nodesets T_z for each point $z \in P$: $(u,v)^u$ and $(v,u)^u$ are in T_z if, and only if, $u = z$. Vertex (z,z) is also included in T_z.
- Only arcs between vertices belonging to different nodesets are copied from the original GTSP, with corresponding costs. Here, the costs between $(x,y)^x$ and $(x,y)^y$ are zero. Note that, for example, an arc from (x,y) to (x,z) in the original GTSP may be transferred to an arc from $(x,y)^x$ to $(x,z)^z$.

This way we again obtain a bi-directed graph. Now, we have a GTSP without overlapping nodesets. A feasible solution of this GTSP has to take at least one vertex from each nodeset. Again, it is easy to verify that corresponding feasible solution can be found with the same total costs.

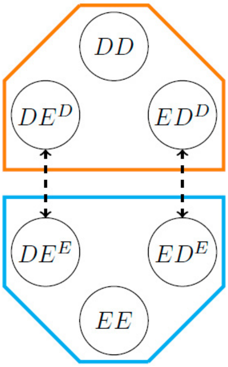

Fig. 8.3. Illustration of result of reduction from GTSP to GTSP without overlapping node sets, where vertex X Y^X corresponds to $(x, y)^x$. The dashed lines have costs zero.

To explain the reduction from GTSP without overlapping nodesets to a TSP, we use the approach of Noon and Bean.[31]

For this TSP, we introduce an arbitrary ordering of the vertices in a nodeset S_i: $i_0, i_1, \dots, i_{|S_i|-1}$. (Note that each i_j is a configuration (x, y).) Next, we introduce new arcs.

- Create in each nodeset a single, directed cycle according to the given ordering, i.e., $i_0, i_1, \dots, i_{|S_i|-1}, i_0$, with the cost of each arc equal to zero.
- Define the costs of directed arcs between any two nodeset as follows: the cost of the arc from $i_j \in S_i$ to $k_l \in S_k$ in the created TSP-instance is set equal to the cost of the arc from $i_{j-1 \left(\text{mod } |S_i|\right)}$ to k_l in the original GTSP-instance.

Now, take an arbitrary solution of our GTSP without overlapping nodesets, say $r, i_j, k_l, \dots, p_q, r$. This then corresponds to the solution $r, i_j, i_{j+1}, \dots, i_{j-1}, k_l, k_{l+1}, \dots, k_{l-1}, p_q, p_{q+1}, \dots, p_{q-1}, r$, where all indices of i, k, and p are modulo $|S_i|$, $\left|S_k\right|$, and $\left|S_p\right|$, respectively. This has the same total cost. To enforce that optimal solutions of the TSP are of this type (meaning that optimal tours visit all nodes of a nodeset before visiting nodes of another nodeset), we add a very large cost β to each directed arc between nodesets. This means that optimal tours will have to visit each nodeset just once and will have a total cost strictly less than $n\beta$. These solutions correspond to optimal solutions of our GTSP without overlapping nodesets. In sum, we proved the following theorem:

Theorem 2
Any instance of 2TSP-MS can be transformed into an instance of asymmetric TSP. An optimal solution of the created instance of TSP corresponds to an optimal solution of the instance of 2TSP-MS, and vice versa.

8.4. Mutual support gap

In this section, we consider the inefficiency caused by demanding mutual support. In order to quantify this inefficiency we define the mutual support gap. Here, an instance of 2TSP is a set of points $P \subset \mathbb{R}^2$ and a root $r \in \mathbb{R}^2$. An instance of 2TSP-MS in addition specifies the maximum allowed distance R between the two drones. For a given P, r, and R, let $Opt_{2TSP-MS}(P,r,R)$ be the optimal value of 2TSP-MS, and let $Opt_{2TSP}(P,r)$ be the optimal value of 2TSP. Note that, in 2TSP, both drones can fly at maximum speed at any point in time. We define $MSG(P,r,R)$, the mutual support gap, as

$$MSG(P,r,R) = \frac{Opt_{2TSP-MS}(P,r,R)}{Opt_{2TSP}(P,r)}.$$

We will first study this quantity theoretically.

Theorem 3
For any instance $P \subset \mathbb{R}^2, r \in \mathbb{R}^2, R \in \mathbb{R}$, we have

$$1 \leq MSG(P,r,R) \leq 2.$$

Proof: First, note that we can adjust any solution of 2TSP-MS to a solution of 2TSP by setting all speeds equal to the maximum speed. Since this will increase the time spent, we have

$$Opt_{2TSP}(P,r) \leq Opt_{2TSP-MS}(P,r,R).$$

On the other hand, a feasible solution for 2TSP-MS is to simultaneously fly a TSP-tour at maximum speed. Let $Opt_{TSP}(P,r)$ be the optimal value of TSP on $P \cup \{r\}$. Moreover, since the total length of the tours in the optimal solution for 2TSP is at most $2Opt_{2TSP}(P,r)$, we have

$$Opt_{2TSP-MS}(P,r,R) \leq Opt_{TSP}(P,r) \leq 2Opt_{2TSP}(P,r).$$

∎

To see that our upper bound of 2 is tight, consider the instance in Figure 8.4. Here, we have a root r, and points A and B. Furthermore, we have $d(r, A) = d(r, B) = \alpha$, and $d(A, B) = 2\alpha$, where $\alpha > R$. In the optimal solution of 2TSP-MS, the two drones will fly together from r to A, from A to B, and from B to r. The value of this solution equals 4α. On the other hand, in the optimal solution of 2TSP, one of the drones visits A, while the other visits B. The value of this solution is equal to 2α.

Fig. 8.4. Instance with mutual support gap 2.

We also investigated the magnitude of the mutual support gap for some real-life instances, provided by the Brazilian Air Force. One of these instances, and the optimal solutions for 2TSP-MS and 2TSP are illustrated in Figure 8.5 and Figure 8.6, respectively. To clarify, the solution for 2TSP-MS in Figure 8.5 uses configurations (9,9), (6,5), and (6,4) consecutively. Hence, the first drone waits at point 6 to sustain mutual support. In Figure 8.6, the first drone primarily focuses on visiting the point farthest from the depot, to minimise the maximum time a drone is flying. It is also clear that mutual support is not provided here. The mutual support gap for this instance is approximately 1.234, i.e., the time taken by the solution of 2TSP-MS is 23.4% more than the time taken by the solution of 2TSP.

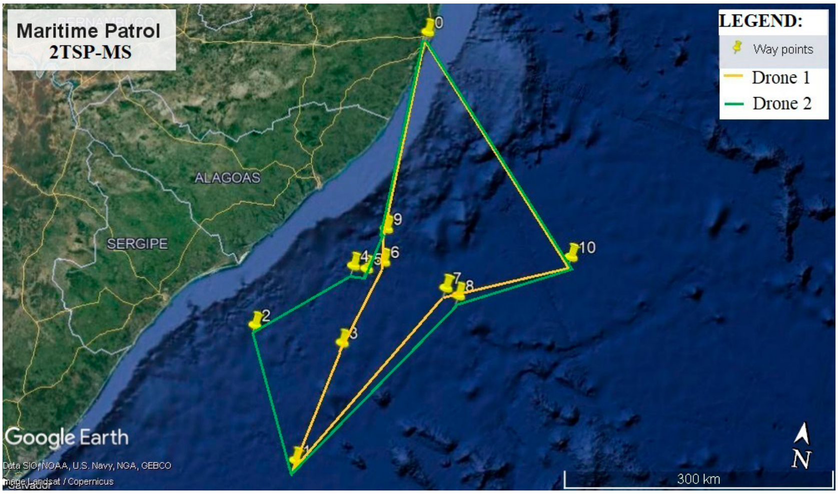

Fig. 8.5. Solution 2TSP-MS. Here, drone 2 waits at point 6, while drone 2 flies from point 5 to point 4.

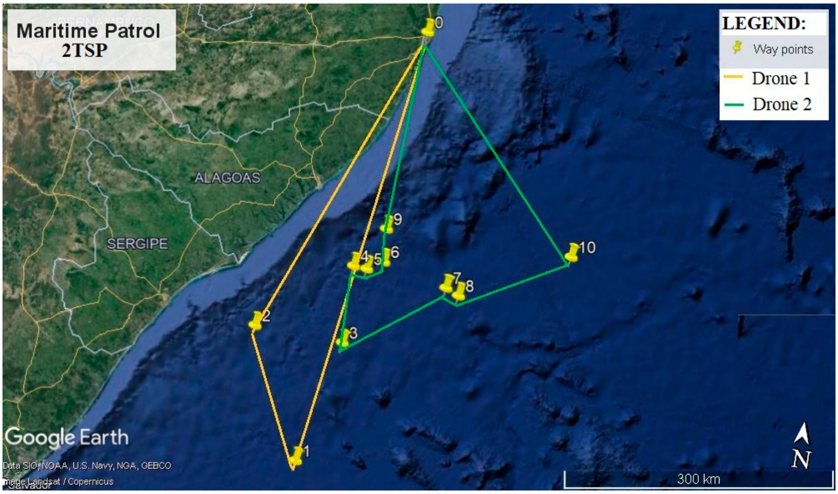

Fig. 8.6. Solution 2TSP.

Naturally, the mutual support gap of an instance relies heavily on geographical features. If the points are located in a "wide" area, like in Figure 8.4, and if the number of isolated points is large, numerical experiments show that the mutual support gap can be as large as 1.5. On the other hand, if the points are "close" to each other, and the number of configurations is large, the mutual support gap can be as low as 1.05.

8.5. Considerations for more drones

So far, we only considered our routing problem for two drones. In this section, we discuss which issues should be considered when dealing with more than two drones.

In Section 8.2, we created a configuration for each pair of points $x, y \in P$, possibly with $x = y$, with $d(x, y) \leq R$, describing the location of each of the drones. This definition naturally generalises to k drones. Whether a certain configuration is created depends on how you define the need for mutual support. For example, if you have three drones, you might define mutual support as having at least one other drone within distance R. Alternatively, you could require that all other drones are within distance R. Depending on your definition of mutual support, it is clear which configurations should be included in the reduction to GTSP. Now, we will discuss two important issues that pop up in the reduction.

A first consideration is the size of the created instance. In the original reduction, we created at most n^2 configurations. If we have k drones, this will lead to creating at most n^k configurations. Hence, the number of configurations grows exponentially in the number of drones. This limits the scalability of our approach. On the other hand, it might not be sensible to use this generalisation if the number of drones is large. We are still assuming that the drones arrive at their destinations simultaneously. Already for four drones, this assumption can increase the time spent significantly. In this case, it might be more convenient to partition the set of points and assign a pair of drones to each of the parts. However, it can be quite complicated to find the optimal partition.

Another issue is related to the result of Theorem 1. There, we showed that the drones can move between any pair of configurations spending the least amount of time possible, while guaranteeing mutual support. This is still true for more than two drones if we define mutual support as having all other drones within a distance of R. As we will show, this result does not always hold for the case with at least three drones if we define mutual support as having at least one other drone within distance R. Here, it is always possible to move the drones from points $\{A, B, C\}$ to points $\{D, E, F\}$ and guarantee mutual support, but this might not be the most efficient movement between the two sets.

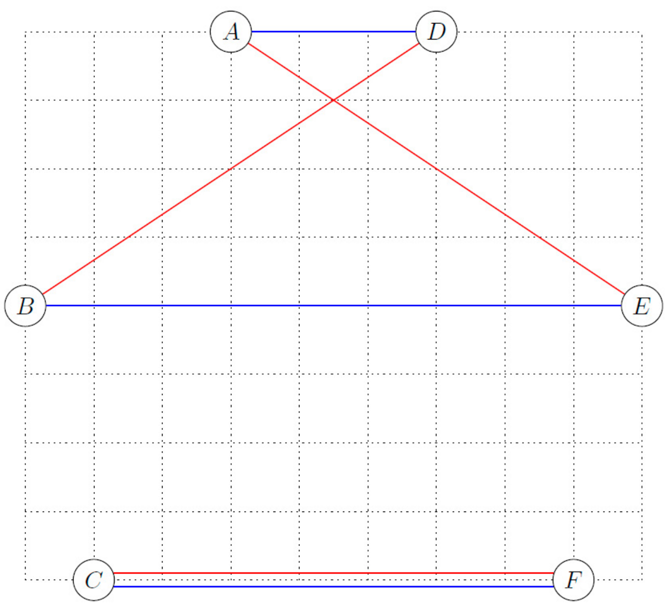

Fig. 8.7. Instance, with R = 5, showing that it might not be possible to move from any permutation of {A, B, C} to any permutation of {D, E, F} in the fastest way while guaranteeing mutual support.

To see this, consider the example in Figure 8.7, where $R = 5$. If the drones move from configuration (A, B, C) to configuration (D, E, F) (blue lines), we guarantee mutual support. This movement takes $9/S$ time. If we move from configuration (A, B, C) to configuration (E, D, F) (red lines), we only need $2\sqrt{13}/S$ time. However, this move is not allowed since the third (bottom) drone is not supported by either the first or the second drone for some period of time.

This example shows that we cannot move between any pair of configurations anymore. This will not lead to infeasible instances, since it is always possible to move between any pair of subsets of points, while maintaining mutual support. To check whether a certain movement is allowed, we have to solve three quadratic inequalities. For $k > 2$ drones, we need to solve $\binom{k}{2}$ quadratic inequalities to verify if a movement between two configurations is allowed.

8.6. Conclusion

Drones have been used for decades in maritime patrol by several countries.[32] Although many advantages are obtained from their use, drones may require special care depending on the type and risk of the mission. Hence, mutual support can be a way to ensure the fulfilment of the mission. In this paper we prove, using the notion of configurations, that a routing problem including mutual support can be modelled as a generalised travelling salesman problem (GTSP), and can be transformed into a regular TSP. We also examined the costs of operating with mutual support and compared it to the costs of using separate drones that detect and identify vessels. This gap can be as big as a factor 2. However, in instances with a large number of configurations, i.e., possibilities to simultaneously visit distinct points, the gap can be as low as a factor 1.05.

Modelling this problem with more than two drones was also addressed. With k drones, it would create at most n^k configurations, so the number of configurations grows exponentially in the number of drones. Hence, increasing the number of drones considerably increases the computational cost for calculating the optimal solution, and the total mission time (total cost). From an operational perspective, when the drone quantity is an even number and the mutual support condition can be fulfilled by a pair of drones, one should consider the possibility of planning the maritime patrol by dividing the surveillance area so that it is patrolled in pairs of drones.

Notes

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