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Versal Unfolding: How a Specific Folding Can Turn Crease and Tear into Transversal Notions

Preamble

> "*L'enfant naît avec vingt-deux plis. Il s'agit de les déplier. La vie de l'homme est alors complète. Sous cette forme il meurt. Il ne lui reste aucun pli à défaire.*"
> "The child is born with twenty-two folds. What needs to be done is to undo the folds. Then the life of the man will be complete. This is the state in which he dies: nothing is left to unfold."

Henri Michaux, *Au Pays de la magie*, 1941

> "A work of folding and unfolding in which every element becomes always the fold of another in a series that knows not point of rest."

Stephen Heath, *Post-structuralist Joyce: Essays from the French*, 1984

From a mathematical perspective, folds and tears of paper may be considered dual notions: there is a strong relationship between the fold of a material and the expectation of a possible catastrophic tear. In this paper we recall a simple and concrete experiment that took the form of performance art. It revealed that, using only a few sheets of paper, while handling it carefully through folding, it is possible to avoid the pairing between folds and tears. To carry out its analysis, we will first review some aspects of the mathematical theory of singularities from René Thom's original viewpoint. Throughout its course we will touch on complex notions such as versal unfolding[1] and transversality[2] and complex texts – from James Joyce to Stéphane Mallarmé. Our conclusion will take the form of an additional performance to be shared with our willing reader.

This paper is concerned with a performance, which could be construed as a simple or naive form of scientific experiment in materials science. This experiment will be the starting point of a highly non-linear random walk through seemingly unrelated fields of science, literature and the arts. Our paper is not intended to be strictly scientific in essence or form, rather it should be regarded a performance in itself, one in which two authors, as two sides of a sheet of paper, share ideas that resonate through a process of folding and unfolding.

This paper is organized as follow. We begin with an account of a performance of ours, which took place at a workshop organized by the research group GDR Mephys at ESPCI. On that occasion we presented a folding process – one we refer to by a *paper junction* – which was invented for this purpose. It allows us to assemble a number of sheets of A4 paper to make a longer strip of paper. During the performance the robustness of this junction was tested in a way that involves the bodies of the two performing artists. We then follow with a motivation for our choice of performance. We recount additional experiments involving our paper junction, which we conducted in order to assess its robustness and its optimal usage. In the third part that follows, we gradually turn reading itself into a performative act, by folding in concepts of pure mathematics one after the other in the manner of accordion folds, in resonance with concepts that underlined our performance. We then continue with the fourth part of this paper, where we unfold and fold several sources that evoke the fold as a metaphor. We jointly discuss Joyce's *Finnegans Wake*, a possible principle of science creativity and Stéphane Mallarmé's poem *un coup de dé jamais n'abolira le hazard*, that epitomize our conception of the fold as what oscillates between metaphor and matter, between inside and outside, between the singular and the general. In the final part of this article, we therefore conclude with a performance that utilizes the very last figure in this paper – an actual folding act of the paper this article is printed on by readers themselves.

1 The notion of versal unfolding is a fundamental tool in the mathematical theory of singularities: a possibly singular object might be difficult to understand when considered *per se*, but is better understood when embedded in a family of often simpler configurations.

2 Transversality is an important notion in differential topology (Thom 1958). Two transverse submanifolds (e.g. curves drawn on a sphere or surfaces embedded in a three-dimensional euclidean space) intersect transversely if at every component of intersection (e.g. points resp. curves), their corresponding tangent spaces at that point generate the tangent space of the embedding manifold at that component.

1 A paper performance

Earlier this year we were invited to perform an art and science act titled *Onto the fold* as part of the workshop *Folding and Creasing of Thin Plate Structures* held at the ESPCI.[3] In what follows we will discuss this somewhat unusual contribution to a scientific conference.

Two performers get in front of an audience to prepare for what is about to happen. The experiment is quite simple. Its simplicity is demonstrated by making the process as straightforward and participatory as possible. Throughout the performance the weight of the two performers is used to test the strength of a paper junction designed especially for the performance. Its folding process can be seen in fig. 1. We refer to the resulting paper junction by the term *Peysson Junction*. The performance demonstrates how one can generate a firm join by folding together two sheets of standard A4 paper that combine through a series of steps to create a longer strip of paper. Joining together the two sheets does not involve any binding material (glue, thread, etc.) nor does it require any paper cutting. It only requires folding.

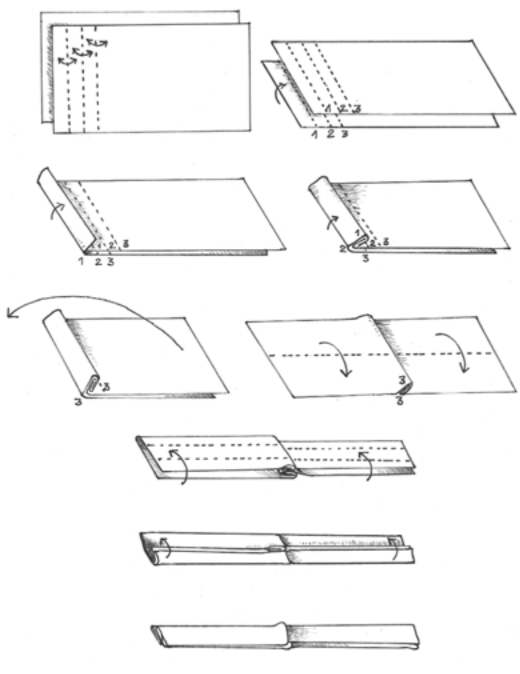

Fig. 1: Junction making guide

3 A one-day workshop organized by the research group GDR MePhy and Keith Seffen (Department of Engineering, University of Cambridge) held at the ESPCI, Paris, March 26, 2015. See Ferrand/Peysson 2015.

As the performance begins, four members of the audience are each handed a sheet of A4 paper. Each is asked to carry out a *J3 folding* defined as follows: folding comprises three equidistant folds (approximately 3 cm wide) that run parallel to the short edge of the paper. The four sheets of folded paper are then collected; two of them are now assembled according to the subsequent steps in fig. 1. Each two-sheet ensemble gets folded in half along the middle line that runs parallel the long edge. Each then gets evenly folded in three along two middle lines that run parallel the same long edge. We refer to the latter two steps of the sequence as a *P2-3 folding*. The resulting fold at this point is called a J3-P2-3 junction – none other than a Peysson junction.

Once each Peysson Junction is ready, force is applied to either end of the paper-strip, as each performer, grabbing hold of one end, starts leaning backwards. As the angle increases between the horizontal axis of the paper-strip and the axis of alignment of each performer's body, so does the intensity of the force applied at each end. The performers' bodies and the strip of paper together form a system, whose internal tension is gradually increasing. The experiment goes on up until its inevitable failure – one or both performers eventually fall to the ground.

There are a couple possible scenarios to explain the final rupture: either it occurs as the paper splits apart somewhere away from the fold, or it happens at the join. In the latter case, either the fold at the join comes undone as one sheet of paper slides off the other, or the paper splits apart along one or more of the fold-lines (as a result of a possible susceptibility induced by folding).

During the performance at the ESPCI it was the second author who took the fall. In that instance, the paper splint apart away from the join. It was the paper itself that proved weaker, as its fibers came apart. The folded junction on the other hand remained intact. The rupture ran perpendicular to the long axis of the paper-strip. The split was similar to what one would eventually expect to get, from a sustained simultaneous pull at two short edges of a single sheet of A4 paper. The result is shown in fig. 3.

2 A paper trail from the discrete to the continuous, and vice-versa

2.1 Foundations

The authors' paper performance has been designed with the intention of bringing together two pairs of concepts: 'discrete/continuous' as the first pair, and 'fold/tear' as the second. It differs from Origami, in that Origami typically engages a single sheet of paper, a fact, which renders a cohesion and resistance to the resulting object – a complex three-dimensional shape, which can possibly maintain some degree of flexibility. Either way, the result has a lower mechanical strength compared to that of the original sheet of paper (or sheets of paper), since folding introduces greater fragility.

Let us begin with discretization and continuity. Consider a standard sheet of A4 paper. Its overall area determines a unit of discretization – the discretization, for instance, of continuous text as it is set in print format.[4] Through our performance we examined the possibility of starting from a discrete set of objects – a few sheets of paper – so as to end with a design of a continuous object – a continuous strip composed of joined paper sheets. We wished to test whether we could generate good longitudinal resistance in such a way that weakness does result from juncture points. We wanted to see whether structure as a whole could be made ignorant of underlying discreteness. Our goal, through a process of trial and error, was to devise an overall folded structure to be much longer but at the same time as strong as the original sheet of paper itself, keeping in mind there is an obvious force threshold beyond which our paper strip will break, and our paper-system will get discretized again.

The dual pair fold-tear forms the second axis of our analysis. The fold is often used when one wants to rip (but not cut) paper along a certain line. A fold introduces a discontinuity in curvature to the otherwise flat, continuous surface of the sheet of paper. Energy, derived from torque applied to the planes on each side of the folded sheet of paper, is focused along the line of the fold – the line of flatness discontinuity.[5] Tear, as a result of ripping, at a given point along the ling of the fold, occurs when this energy exceeds a threshold. Ripping begins at the point where the fold line meets the border of the sheet of paper. A gradual separation into two parts then occurs. In this example, fold and tear appear as dual concepts: where a crease is, tearing will be.

2.2 Additional experiments

Our performance was supplemented by a series of resistance tests conducted in the laboratory so as to confirm the reproducibility of the art/science experiment. Five different tests were carried as follows (each repeated several times):

Test 1. Two sheets were joined together through a J3-P2-3 Peysson Junction (as during performance, see fig. 1). The resulting strip of paper was tested against sustained pull

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4 Thus, for example, text flow is governed by the logic of content, but the setting of text into printed format requires dividing it in relation to a unit of area – the area of the sheet of paper in question. Compare section 4.3 in this paper.

5 The axis of torque runs perpendicular to the plane of the paper.

till it split apart. The remaining strip of paper – now shorter – was put to the test again till it split apart in its turn. Its remaining bit was then retested – now even shorter – till it broke apart as before. The process continued till the strip of paper got too short to be tested, nevertheless maintaining the J3-P2-3 join. The experiment was repeated three times from the beginning. Results were identical: at the end of each test the resulting tear ran perpendicular to the long axis of the strip, away from the junction, and rather close in fact to one or the other end (it was not possible to predict which end, though). See fig. 2.

Test 2. Three sheets were joined through two J3-P2-3 Peysson junctions. Same positive results as in the first test: the tear ran perpendicular to the long axis of the strip, away from the junction.

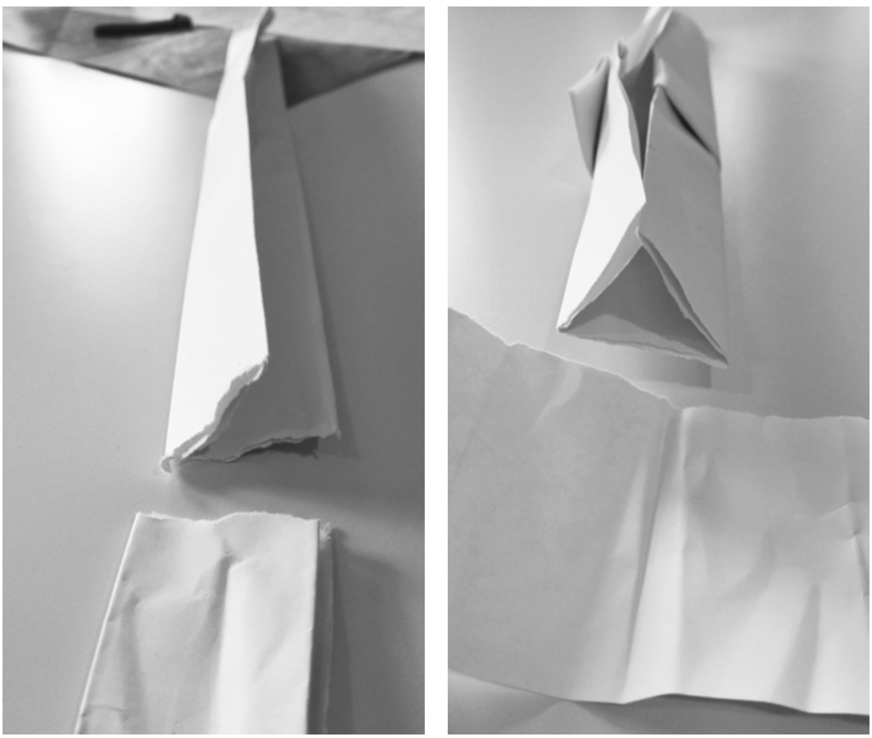

Fig. 2: The strip is torn away from the fold.

Test 3. Two sheets were joined through a *J2-P2-3 Junction*, namely, two folds at the first stage of folding instead of three. The results here were different. The strip of paper split apart along folds at the junction. All repeated tries of this experiments showed the same result: the fold ripped (fig. 3).

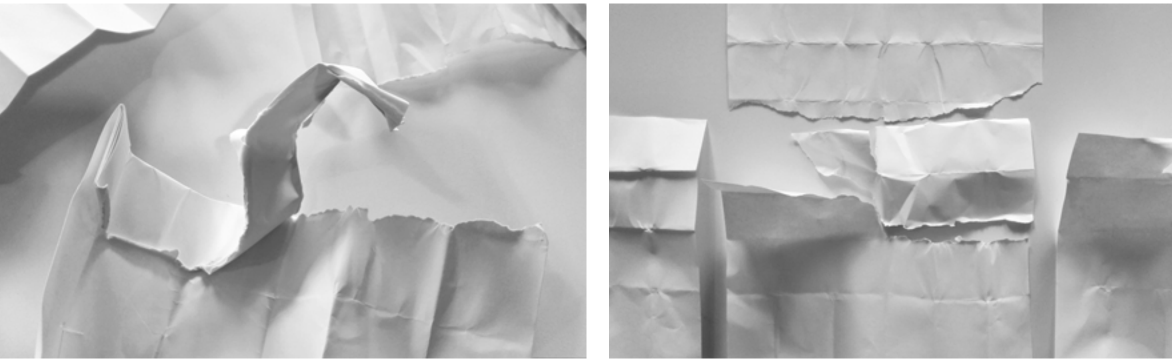

Fig. 3: Weaker junction.

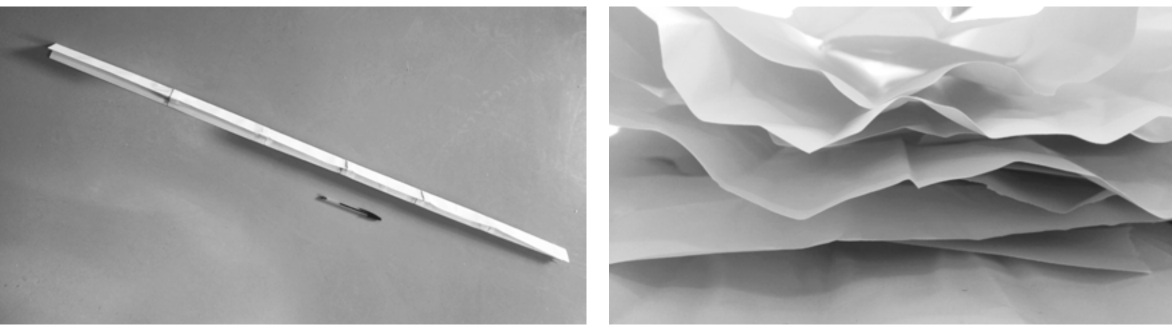

Fig. 4: A longer strip.

Test 4. Two sheets were joined through a J3-P2-3 Junction, only this time the J3 folds were narrower (about 1cm instead of 3cm). The same process of experiment was repeated as in Test 1 with narrower folds. The results were as conclusive as with the first test.

Test 5. More sheets of paper were joined through Peysson Junctions to form an even longer strip of paper, leading to results as good as the first test (see upper image of fig. 4).

Further experiments were conducted varying in paper quality and size, using again a J3-P2-3 Peysson Junction, as in test 1. Variation did not lead to significantly different results. Of course, the use of softer paper (such as paper towels) resulted in the junction coming apart even when little force was employed. A certain degree of paper density seems necessary to ensure junction stays intact. There appears to be a bifurcation threshold, depending on paper density, across which behavior changes drastically. Below the threshold, the two (or more) units of paper come undone by sliding off each other. Above the threshold, the break happens as a result of a tear in one paper unit, away from the junction.

3 Unfolding mathematics with a qualitative twist: Thomism

The tearing of the paper in the above experiments points towards a possible catastrophe, that is, a sudden change in the continuous system. Instead of carrying out a quantitative analysis of paper resistance to stress, we wish to present the reader with qualitative mathematical ideas, and discuss the relevance of folding to mathematics in relation to the paradigms of continuity and discreetness.

The French mathematician René Thom (1928–2002) is associated with two notions of folds and continuity noted above. In 1959 Thom was awarded the Fields Medal for his work in the field of topology[6] (roughly speaking, the study of shapes up to continuous deformations). He later became a major contributor to the related field of singularity theory – the classification of local behavior of various geometrical objects and mappings. He is often associated with *catastrophe theory*, which had its moment in the limelight in the 1970s (more on this in the sequel). Catastrophe theory consists in using ideas from singularity theory to describe the sudden changes that may occur in the qualitative behavior of concrete systems that arise in fields such as biology or economics.[7] It is possible Thom's original ideas were over-interpreted by his followers and epigones.[8] Catastrophe theory was initially not seriously considered; it became a subject of ridicule when it was rather naively adopted by some in order to introduce a little bit of mathematics into the social sciences, as if such proof of rationality was needed to begin with. In fact, catastrophe theory provides a useful conceptual system to analyze phenomena, which were previously understood using more technical, quantitative models. It provides a simple language with which one can interpret and unify disparate phenomena; indeed, Thom was not afraid of simplicity. For him, non-trivial concepts were all the more interesting when they could be expressed in a simple, non-technical form.[9] Later on during the 1980s, catastrophe theory was eclipsed by *chaos theory*, not in opposition to catastrophe theory, but partly influenced by it. Thom was something of a free electron in the French mathematical landscape, at a time when the Bourbaki group exerted the most influence. Bourbaki's radically formalist line ran opposite to Thom's ideas.[10] We shall return to this issue in Section 4.2.

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6 He was awarded for his work on cobordism theory.

7 A full account of catastrophe theory can be found in Arnold 1992. Let us briefly cite from Tsatsanis 2012, 217: "[catastrophe theory] has as its goal to classify systems according to their behavior under perturbation. When a natural system is described by a function of state variables, then the perturbations are represented by control parameters on which the function depends. This is how a smooth family of functions arises in the study of natural phenomena, […]" one of its members being a function with critical points.

8 See for example Zeeman 1977 and Smale 1978 for an account of the polemics induced by the popularization of catastrophe theory in the 1970s.

9 Thom signed a disapproving academic report on Jacques Derrida, claiming that his convoluted, difficult language does not hide any deep or subtle thought. See Smith 1992.

10 See for example Thom 1970.

3.1 Ontological anteriority

Of prime importance to us is the *anteriority principle* Thom promoted, namely, the principle of the ontological precedence of the continuum over the discrete.[11] For Thom, discrete objects arise as accidents in the continuum,[12] or they may come about as topological invariants of continuous objects.[13] The anteriority principle reverses the traditional order in which mathematical objects are typically introduced: in the classroom, we are first taught the system of natural and negative numbers (0, ±1, ±2, ±3, …). The system of rational numbers is then introduced,[14] which is then completed to form the system of real numbers or the real line – the basic continuous object, used, for example, to model our intuition of time. Thom proposed to reformulate mathematics from this viewpoint, starting from what to him was the most fundamental object – the line drawn on a chalkboard.[15] Even though he himself admitted his program could not be carried out formally,[16] he did not see this as a failure, as he often expressed a critical view of the formalization of mathematics.[17]

3.2 Unfolding

How does this reformulation, that is, the anteriority principle, as what commences from the continuous, manifest itself in Thom's theory? The concept of *versal unfolding* in singularity theory that we are roughly going to sketch now is partly based on Thom's ideas; it illustrates, as we shall see, the anteriority principle discussed above. Singularity theory attempts to understand a possibly complicated singular (as opposed to regular) object by embedding it (or unfolding it) in a continuous family of objects of a similar, albeit simpler, nature.[18] By doing so in a suitable manner, one often observes a natural stratification of the parameter space of this family, which reflects the complication of the object corresponding to a specific value of the parameter. The stratification often exhibits a rich structure, which may give some important information on the original singular object under study. An important idea in singularity theory is that one may identify two apparently different objects if one can pass from one to the other using a transformation, which preserves attributes of the objects in question. In many cases, modulo those identifications, one can find an unfolding, which is at the same time not

11 Thom 1992.

12 Such as an arbitrarily marked point on a line, or the moment when the long hand of a clock lands at '12'.

13 Such as the number of turns of a rope around a mooring bollard.

14 By 'rational number' we mean a positive or negative fraction such as 3/4 or 5/3.

15 Thom 1990. See also Thom 1992.

16 As remarked in Thom 1992.

17 Thom actually promoted the idea that anything stated rigorously is insignificant. Cf. Thom 1968. See also Salankis 1999, sections 4.3 and 4.4.

18 See Thom 1975, especially chapters 3, 4 and 5.

too complicated but general enough to cover all possible qualitative configurations. This is a versal unfolding.[19]

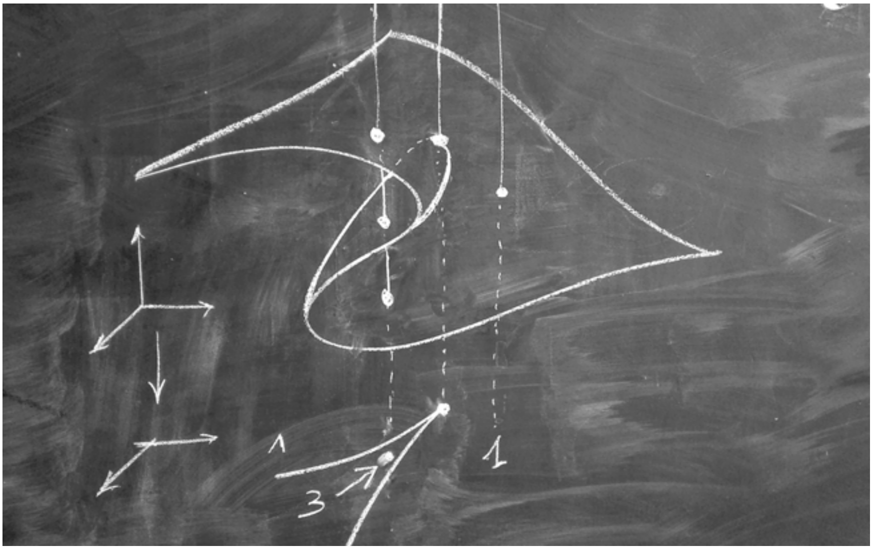

Fig. 5: Fold and cusp.

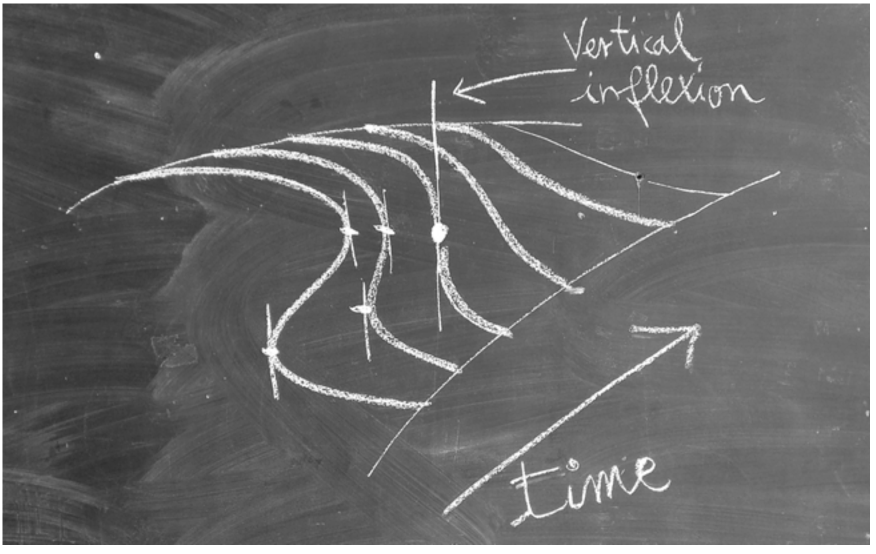

Fig. 6: Unfolding an inflexion.[20]

19 Here we understand 'versal' as in 'universal' or 'transversal'. Arnold 1992 is a good introduction to the mathematical definitions of these concepts.

20 The chalkboard illustrations are a homage to Thom, who praised chalk and blackboard.

3.3 Folds and cusps

We now come to a smooth model of a fold ubiquitous in catastrophe theory, which will provide at the same time an illustration of the concept of unfolding. In fig. 5 one sees a blackboard drawing of a local model of a regular mapping from a domain of the plane back onto the plane: a local model of *smooth folding*.

The mapping is defined as follows: the points of the domain are first embedded in three-dimensional space following the picture, and then mapped back to the plane using the vertical projection. Some points in the plane below are the image of exactly one point, while others, as a direct result of folding, are the image of three different points of the embedded domain above. This dual characterization of points defines two regions in the image below – the region of single-source points, and the region of triple-source points. Exceptional points lie at the border between these two regions. The exceptional points form a border curve – a cusped border curve[21] – consisting of two branches, which meet at a special point – a singular point – in the image plane. The set of points in the domain that map onto the cusped curve form a smooth curve.[22]

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21 A *cusp* is a singular point of a curve, which locally, when located at the origin of the (x,y)-plane, its neighborhood looks like that of the curve $\{y^2 = x^3\}$. The fact that a cusp is created during this projection is noticeable since, in general, the projection of a smooth spatial curve does not have any cusps, but is itself a smooth curve in the plane, possibly with some isolated self-intersections, called *nodes* (i.e. a point that locally looks like the intersection point of two lines). See Hartshorne 1997, 310. When one works over the complex numbers and examines projections of complex surfaces (of degree > 2) into the plane, then every border curve generically must have cusps and nodes (the "border curve" in mathematical jargon is called *branch curve*, being the projection of the *ramification curve*. The ramification curve is, one might say, the curve along which the surface is folded with respect to the projection). For a survey on branched coverings and the special relations between the nodes and the cusps of a branch curve, see Friedman/Leyenson 2011.

22 Note that the number of points in the pre-image of a point along the cusped curve equals two. A more formal description of fig. 5 is as follows. It represents a folded surface, embedded in three-dimensional Euclidean space, R^3, and then mapped back into the plane by projecting in the vertical direction. The surface is parameterized by a domain in the plane, for which we will use (t, s) as coordinates (we assume that this domain is a neighborhood of the origin (0,0)). We will map this domain in R^3 using the following formula:

$$(t,s) \rightarrow (x = t,\ y = s^3 - ts,\ z = s)$$

The image of the domain under this map is the folded surface in the picture. The real fold is actually achieved when this surface is mapped to the plane by the vertical 'forget z' projection, so that the map from the domain to the image plane takes the following form:

$$f: (t,s) \rightarrow (x = t,\ y = s^3 - ts)$$

Its differential is not of maximal rank 2 if and only if $3s^2 = t$, i.e. on the parabola (note that when this relation between s and t is substituted for the equation of the surface, we obtain the following curve in R^3, parameterized by s: $s \rightarrow (3s^2, -2s^3, s)$, which would be the *smooth* ramification curve). Outside this curve, it is a local diffeomorphism. The curve is mapped by f onto another curve parameterized by s as follows

$$s \rightarrow (x = 3s^2,\ y = -2s^3).$$

This curve has a singular point at $s = 0$, called *a cusp*. In the image plane, this curve separates the regions where a point has one or three pre-images.

In thermodynamics a similar picture gives a precise account of the transition from metastable states to stable states. Note that this and was well known long before catastrophe theory came about.[23] Many prototypical examples from catastrophe theory can be examined through the metastable-stable thermo-dynamical paradigm. These examples arise in biology, economics or the social sciences, in contexts in which, at least on a qualitative level, a model involving the minimization of some energy can be introduced.

3.4 Unfolding an inflexion

The mapping between two-dimensional planes, presented in fig. 5, introduces a model of cusp singularity. However, the very same picture also corresponds to an unfolding of a vertical inflexion, as can be seen in fig. 6.

The folded surface (the above domain) can be foliated into curved lines that are parameterized by the real line,[24] as if one is looking at a movie, which tells the story of a curve that develops and changes over time. The curve with the vertical inflexion is singular in the sense that it is qualitively isolated within this family: when it is perturbed the inflexion disappears. Perturbed in one direction, two distinct bumps appear, perturbed in the other direction, there are none. Either way, the perturbed curves run perpendicularly to the vertical direction. The vertical inflection corresponds to the singular moment, where two bumps meet and annihilate each other. Here lies the idea that underlies unfolding: a singular object often corresponds to a limit process, as a boundary or a wall crossing – an accident in continuity. To be understood as such, a singular object cannot be studied on its own, one has to include it in a family of neighboring objects so it could be deformed. From this viewpoint, singularity – and hence also a discrete object, viewed as singular – becomes a relative concept.[25]

23 See for example geometrical methods in thermodynamics introduced by Gibbs 150 years ago.

24 The same piece of surface embedded in R^3 can be used to understand the deformation of a vertical inflexion. Using the notation of footnote 22, for any fixed value of the parameter t, which we should now consider as representing time, we can cut a t time-slice of the surface to get a curve parameterized by s in the (y, z) plane. For $t = 0$, this curve $s \rightarrow (y = s^3, z = s)$ has a vertical inflexion: the vertical direction (z axis) is tangent to this curve, but the curve goes through this tangent at the origin. As t varies, the curve is perturbed. For t positive, the curve gets two bumps: the inflexion is resolved into two generic (quadratic) tangencies (i.e. one obtains a function with a minimum and a maximum, to which two lines, parallel to the z axis, are tangent). When t is negative, the curve is transversal to the vertical direction and has no tangency with any line parallel to the z axis. The curve at $t = 0$ is singular in the sense that it is the only one of its kind in this family, i.e. it has only one tangency point with the z axis and it is of multiplicity 3. Perturbing it in both directions gives rise to a curve with simpler tangencies, if any. This family of curves is called an unfolding. The union of all those curves for all possible values of t forms our original folded surface.

3.5 Smooth folding versus paper folding

To make the theory precise, these considerations should be developed in the context of smooth objects.[26] Singularity theories can be applied to describe the physics underlying the experiment leading to catastrophe – the disruption in continuity corresponding to the tearing of our paper strip system. The folds (or curves) involved would then be rather abstract: they would lie within energy levels in some complicated high-dimensional phase space. Nevertheless, one should not confuse this approach, where the idea of folding is abstractly hidden, with our simple paper strip system – a concrete folded sheet of paper.

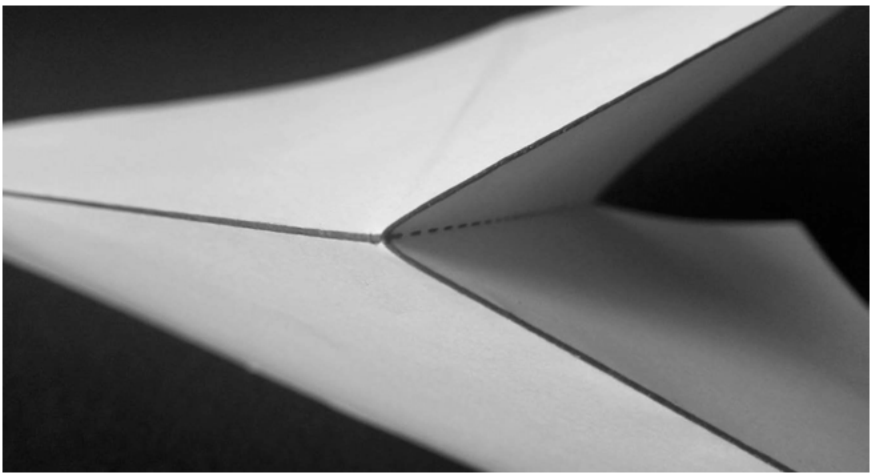

Fig. 7: Flat folding.

In reference to smooth folding as in fig. 5, it is important to note, that if we were to try to realize a similar cusp singularity in an actual model, we would need for that purpose a plane domain made out of an elastic, stretchable material. A sheet of paper is not a good candidate, as it exhibits little plasticity in this regard. A paper model analogue of cusp singularity will hence look slightly differently. It can be found in the simple singularities that occur in the planar folding of a sheet of paper (as when one folds a city map, see fig. 7). The fold lines of a smooth model are replaced by line segments, along which the sheet of paper is creased. Cusp singularities would correspond to the points where

25 We would like to draw attention to Gilles Châtelet (1944–99), a highly original philosopher and pamphleteer, who in *L'enchantement du virtuel,* Châtelet 2010, presented an interesting analysis of an idea closely related to that of unfolding. In the context of his work, 'virtual' should be understood similarly to the principle of virtual work in mechanics, where the static equilibrium of a system (for instance a physical structure such as a bridge) is embedded in some space of possible virtual deformations of its structure. Also cf. Salanskis 2012 for a discussion on the epistemology of Thom's catastrophe theory.

26 Using technical terms, smooth objects are continuous and 'sufficiently differentiable' in a sense appropriate to the given mathematical context.

the line segments meet in the simplest possible way, namely, instances where four line segments meet. One can prove that non-plasticity of paper induces a constraint around each cusp singularity: opposite angles must add up to a flat angle. More complicated non-generic singularities, where a larger (even) number of segments meet, may occur, but one can check experimentally that they do not occur in random flat paper folding.

Nevertheless, complex non-generic singularities are of interest when one designs an origami folding. It would be interesting to further develop its analogy to the classical smooth theory of singularities, in order to understand, for example, how these more complex singularities can be considered a superposition of simpler ones, and how a small perturbation may induce a resolution of the complex configuration into a collection of simpler configurations. To the best of our knowledge, such program has not been carried out yet.

In fact, the smooth setting, which at first glimpse appears more complicated and abstract than simple paper folding, offers many advantages and is quite flexible, once notions of differential geometry are introduced. However, a likely paper folding analogue promises to have more of a discrete flavor: even though a sheet of paper is a continuous media, the resulting pattern of creases is a graph whose edges are segments of straight lines, which is completely determined by discrete set of data.[27]

4 Convoluted literature

In the former two sections, the fold presented the possibility of bridging the distinction and hierarchy between discrete and continuous. It submits itself promptly as a philosophical, as will be discussed shortly in section 4.3. This is also the case with contemporary art as in Roy Ascott's *La Plissure du texte*, with contemporary music as in Pierre Boulez's *Pli selon pli, portrait de Mallarmé*, or with literature as in Joyce's *Finnegans Wake*. In this section "twisted [and] intricately folded" will also mean *convoluted*. Accordingly, in *Finnegans Wake*, we find convoluted characters in folds. As we unfold what the convolution has intricately folded, we reach a higher viewpoint. It is no longer considered speculative nowadays to suggest that creative thinking differs profoundly from Aristotelian logic or a Boolean hypothetico-deductive process. Syncretic thinking, as clearly explained by Anton Ehrenzweig in *The Hidden Order of Art*,[28] is an intuitive and unconscious critical process,

27 In the smooth setting, the curves along which the surface is folded (and especially their image when projecting into the plane, i.e. the branch curves) gives rise to combinatorial data regarding the surface itself so that one can reconstruct the covering using that data together with some additional geometrical data. This is the subject of Chisini's conjecture, proved in Kulikov 1999; Kulikov 2008. See also Friedman/Leyenson 2011.

28 Ehrenzweig 1967.

which can give us access to a more powerful vision of complexity. Thinking in terms of convolution offers a modality for this function of thinking. Scientific creativity may come under it, as it requires researchers to think differently as they pretend to when writing their formal articles. We therefore consider how this convoluted viewpoint could provide a different perspective on other fields of the written domain, such as philosophy or poetry.

4.1 Finnegans Wake

In his essay *Ambiviolences: Notes for reading Joyce*[29], Stephen Heath observed that, in interpreting Joyce's *Finnegans Wake*, critics took to two opposite routes. According to one line of interpretation, the text was regarded as absurd and unreadable, while according to the other it was deemed an enigmatic message to be decrypted.[30] Heath himself rejected both approaches: the first as it *a priori* denied the prospect of opening itself up to many fragments of meaning, the second as it missed the creative reading the text calls for. The second approach, which supposes a hidden message, assumes the text to have an immanent unity and continuity – a reading, which Heath believes to be too restrictive. It is rather that the text offers multiple fragments of meaning, which are traced and retraced, colliding and breaking ceaselessly in a textual play that resists homogenization.[31] There is no single style; instead, there is dissolution into a network of modes of expression. Narrative structure is discontinuous; narration is ambivalent, discouraging an attempt to identify a specific voice. Heath uses the analogy of the fold to describe this peculiar construction: "[...] Finnegans Wake is offered as a permanent *inter*plication, a work of folding and unfolding in which every element becomes always the fold of another in a series that knows no point of rest."[32] *Finnegans Wake* should not be read as assuming a permanent state of meaning, and reading becomes a creative experience for the reader. The novel is an open piece of art. Referring to Umberto Eco, this means it cannot be reduced to a single interpretation.[33] The text is an endless source of meanings. Beyond a purely informative discourse, the folded and refolded text – convoluted – stimulates the expectations of the reader.

29 Heath 1984, 31 – 68.
30 Ibid, 31.
31 Ibid, 31, 32.
32 Ibid, 32 (italics is in the original).
33 Eco 1965, 20.

4.2 Crumpling scientific papers

Against this background of a serial unfolding "that knows no point of rest" and resists any clear, distinct meaning, thereby affording unfolding a flux of meanings, consider scientific papers, which present themselves as examples of an informative text designed to have a clear, linear, discrete reading, without creases or folds, in short, unambiguous (an assertion which itself would be interesting to question). However, creativity in science cannot be reduced to this type of process. The mathematician Henri Poincaré tried to explain in *La Valeur de la science* how much his own discoveries owe to the wandering of thought and daydreaming.[34] Due to present-day academic norms,[35] the production of hard-to-digest, formal, technical literature is experiencing exponential growth. Computer-generated mock-up scientific texts are accepted for publication in scientific journals and conference proceedings.[36] Scientific knowledge is not reducible to the content of scientific texts. Should all scientists disappear from this planet, it is very unlikely science could be reconstructed from archived scientific texts. One could say Joyce's writing is parallel to true scientific practice, in that it gives us a way to perceive the indistinct, the random, the discontinuous in seemingly linear flows.

The Bourbaki project is a singular milestone in the landscape of formal mathematics and scientific writing.[37] Bourbaki was a collective of mathematicians who undertook to provide modern mathematics with a formally sound foundation through a unified approach.[38] Although a great deal of work was done, leading to many useful results, the project remains a fascinating but unattainable ambition. Eighty years since, one is bound to admit this endeavor is beyond human capabilities, as few can absorb mathematics this way. Thom, who was mathematically active as the Bourbaki project reached its zenith, steered away from this circle of ideas. Interestingly, differential calculus, upon which Thom's smooth paradigm is based – a paradigm invoked when one wishes to establish a formal account of the continuum and of singularity theory – was developed by two convoluted writers: Isaac Newton (whose abundant obscure writings extend far beyond science) and Gottfried Wilhelm Leibniz.[39]

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34 Henri Poincaré wrote a whole paper about this process. See Poincaré 1908.

35 The so-called publish-or-perish imperative.

36 Several papers randomly generated by Mathgen were accepted for publication after a peer review process. See *http://thatsmathematics.com/mathgen*

37 Bourbaki 1939.

38 A good introduction to Bourbaki can be found in various works by the French poet Jacques Roubaud. See Roubaud 1997.

39 Leibniz himself uses the fold as a metaphor to a more or less explicit degree: Deleuze famously noted in reference to Leibniz's monadology: "In the labyrinth of the continuous the smallest element is not the point but the fold." See Deleuze 1988, 9.

4.3 Folding mystery

Many authors utilized the fold as a metaphor: Martin Heidegger (the fold of our being), Maurice Merleau-Ponty (the fold as chiasmus or interlacing), Michel Foucault (the fold of in and out), Gilles Deleuze (surface fold), Jacques Derrida (especially in his writing about Joyce, the fold as the impossibility of simple self-identity) and Jean-Luc Marion (fold of the given), to name a few.[40] This goes beyond the scope of this paper; here we will merely mention the work of Quentin Meillassoux[41] on Stéphane Mallarmé's seminal poem *Un Coup de Dés Jamais N'Abolira Le Hasard*.[42] This poem is a landmark of French literature, possibly of the entire literary corpus. Published at the very end of 19th century, it represented a radical rupture: the poem, whose form in print is of primary importance, exhibited an intriguing interplay between the two-dimensional page and its non-linear typographical layout. Its syntactical structure was highly non-standard, in particular in its use (or lack of use) of punctuation. With about 700 words per 11 pages, a vast majority of the paper surface was left blank. The words, printed using various font sizes, were laid out in a succession of clusters, which may evoke foam on the crest of waves. Meaning, if indeed there is any, has remained mysterious for more than a century; the poem has been the subject of countless studies and interpretations. Like in the case of Joyce's *Finnegans Wake*, it has been considered absurd and unreadable by many, or, alternately, a challenging encoded mystery by others. Quentin Meillassoux recently proposed a cryptologic interpretation,[43] largely based on a certain 'discrete' numerology. More precisely, it involves a careful consideration awarded to all words and signs in their respective positions – the most basic 'discrete' elements of text. Meillassoux's precise, quantitative, documented work ignores in effect most of the poem's 'geometry' (layout, varying sizes, global and local blank regions). Meillassoux is nevertheless able to draw rather convincing conclusions from his analysis. Roughly speaking, some numbers are not there by accident; it seems that Mallarmé adopted a hidden structure governed by obscure computations. We note that this discovery, if confirmed, is but one interpretation that, albeit appealing, still disregards more material, visual aspects.[44] We would like to suggest a more 'folded' approach, one possibly based on manipulations (yet to be discovered) of the poem's underlying sheet of paper used here as the two-dimensional

40 For these fold metaphors see Cormann/Laoureux/Piéron 2005.

41 Meillassoux 2011.

42 Mallarmé 1897.

43 Kaplan 2012.

44 Poetry was for Mallarmé a form of a new religion, encompassing other forms of artistic expression: it has its own musicality, and also in this poem, a material and a visual representation. See Meillassoux 2015, 65, 104.

space of poetry.[45] Even if an actual act of folding is abandoned, we feel that the inherent folded nature of those clusters of words – suggesting waves of a rough sea – should be taken into account. It seems to us that a third folded approach should be followed similar to the one suggested by Heath: the words themselves would be folded and unfolded onto each other. This would not substitute the intriguing search for the ultimate (discrete) decryption à la Meillassoux, but would run parallel to it.[46]

Looking at the history of ideas, one observes that the fold, as a metaphor, has often been used in philosophical dialectical contexts. It has been invoked to deal with single and multiple, identity and difference, inside and outside or the constituent and the constituted. The image of the fold appears to be an effective metaphor whenever one contemplates complexity. As evidence, one only need consider its many occurrences in contemporary philosophy. Creative thinking is based on a complex relationship between all manner of rational and irrational thought. In that sense, convoluted images support creative imagination. The writings of Joyce, Thom and Mallarmé invite a creative reading that allows the reader to follow convoluted paths and nebulous themes.

Our existing models of complexity required revision, which prompted the emergence of chaos theory and of the notion of a complex system. Folds, convolutions, windings and labyrinths are all fruitful methodologies that help to structure, represent and rethink the complexity of our world. We followed here some folded dualities of our own, in a discussion starting from a folded paper junction. The fold has often been associated with infinitely divisible continuity. Our physical performance has taken an opposite direction, creating a structural junction from the discrete to the continuous. Our bodies were fully engaged in a dynamic one-dimensional experiment, experiencing the moment when the continuous broke into the discontinuous. The sudden toppling over of one of the performers brought on catastrophe. In the same vein, we presented a patchwork of mental images, dealing with seemingly unrelated subject matters, hoping readers will find their way to continuously combine them.

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45 Mallarmé gave much thought to the layout of book leaves in the folio. Different combinations of paper-sheets suggested rhymes. The book binding process was important to Mallarmé, who considered the fold to be the central abyss, the "discontinuity of meaning," the nonsensical part associated to text signification. The white part of the page he thought of as the void. See Meillassoux 2015, 59.

46 Randomness (*le hasard*) was of great significance to Mallarmé. See Meillassoux 2015, 39.

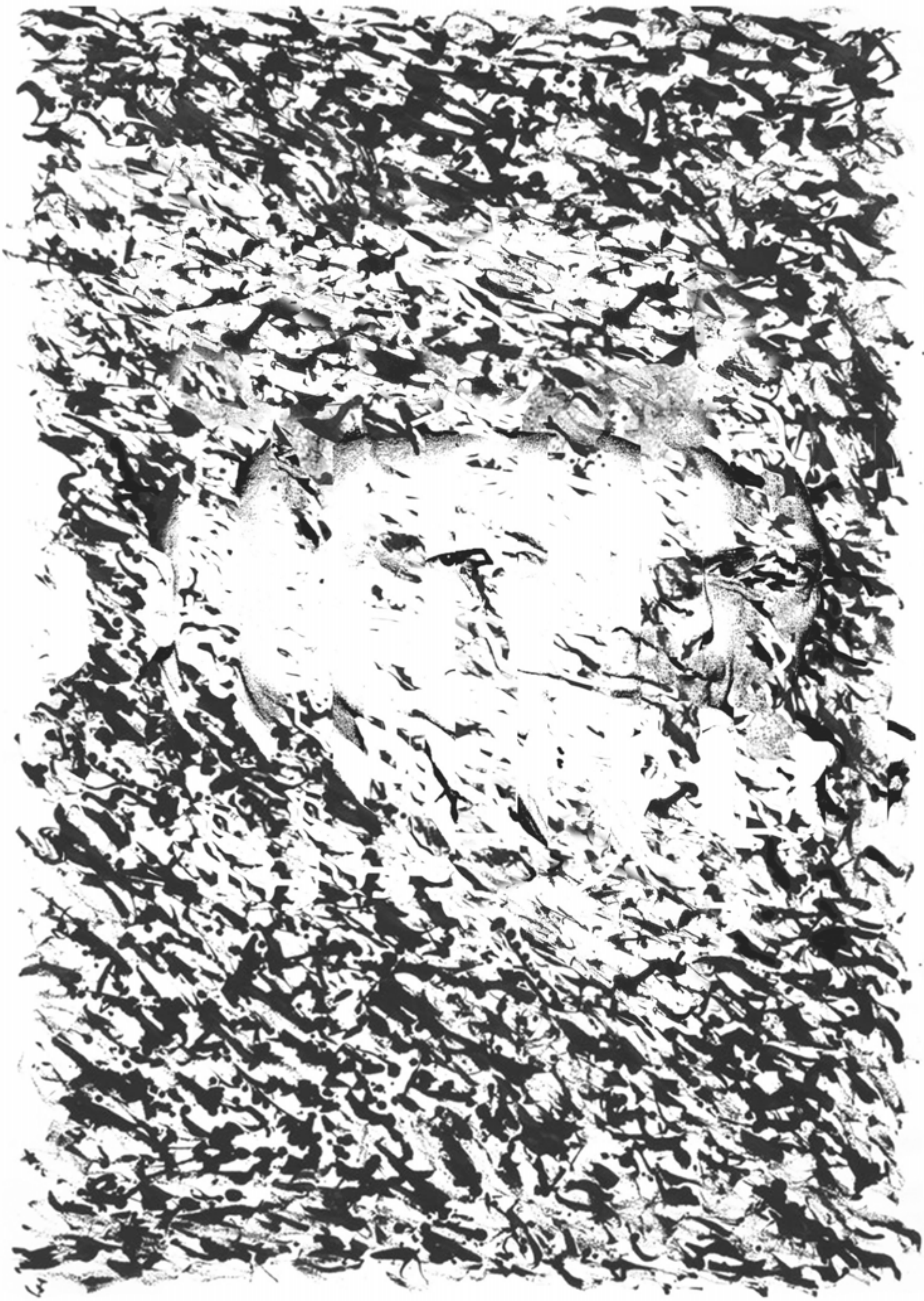
5 Appendix: instruction for a printed performance

The sense of mystery suggested by the labyrinthine convolutions of folds is crucial to their nature. The fold can hide what has been laid down on paper. It offers an effective way to encode messages. We explore this theme here in a proposed folding experiment. We encourage the reader to print and fold the very last figure (fig. 8) following our instructions below. It brings us back full circle to Henri Michaux, the poet and painter cited at the very beginning of this paper, as the experiment refers to a modified (unnamed) ink print of his – this paper's last figure.[47]

Note the markings at the top and bottom of the very last image: points, thin long segments, thick short segments. The reader will need to carefully fold the sheet of paper several times parallel to the long edge. The paper should be folded in 'V' shape (a valley fold) across each line that joins a matching pair of thin long segments – a line that runs parallel to the long-edge of the paper. The resulting fold should stretch across the entire length of the paper from top to bottom. In a similar manner, the paper should be folded in 'Λ' shape (a mountain fold) across each line that joins a matching pair of thick short segments. Once this is done, each thick line should touch a corresponding point – the continuity point. The folds should be carefully flattened. Once all folds are accomplished, the reader will observe the result across the flattened width of the paper.[48]

47 Henri Michaux, Sans titre, ca. 1975, ink on paper (mongrammed at lower right), 74,5 x 105 cm, Belgium, Private Collection. This piece by Henri Michaux will be included in the future in a catalog prepared by Micheline Phankim, Rainer M. Mason and Franck Leibovici. The catalog will present prints of his paintings and drawings, the majority of which feature in private collections.

48 Should the reader find the printed image too small to fold, he or she may download the original at: *http://webusers.imj-prg.fr/~emmanuel.ferrand/fold/*.



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