1. Bodies of Sense-Making

The *Mathematics-Rationality-Human* Continuum

> In what ways do histories of mathematics help us to think about ourselves as human, as rational, as modern? Who is invited to see themselves within these histories and who is excluded from these histories?
> *Sara N. Hottinger, Inventing the Mathematician (2016), 12*

1.1 *Mathematics* as an Image of Thinking

This is a thesis on the epistemic violence of *Mathematics*, which as I wrote in the beginning pages of this thesis, gives rise to the question –what is *Mathematics*? This first chapter is dedicated to this very question. I provide a historical study of *Mathematics* as an Image of Thinking. This exploration has or is structured by two goals: a) to characterize what defines *Mathematics* as an Image of Thinking and b) to historically contextualize these epistemic characteristics and *Mathematics* as an Image of Thinking.

Conceptualizing *Mathematics* as an *Image of Thinking* guides this historical study. Crucially, I do not explore *Mathematics* primarily as an academic discipline or as an accumulation of specific epistemic operations. Rather my objective is to account for everything that *Mathematics* is that exceeds its existence as an academic discipline. So, *Mathematics* here denotes an *Image of Thinking* that permeates all areas of contemporary social life as an epistemic notion and practice. In doing so, I explore how contemporary Western culture is permeated by a specific epistemic concept and practice of *Mathematics*, which in turn relates to the notion of *Rationality* as well as the notion of *the Human*. I understand *Mathematics, Rationality*, and *the Human* as both notions and practices of thinking by building on the concept of an *Image of Thought* as put forth by Gilles Deleuze (Deleuze 1997, 169). My variation of his term emphasizes that I am exploring the notion of a practice as a practice itself. It emphasizes thinking as a doing, which I believe aligns with the nature of Deleuze's notion and his work more generally as Deleuze's notion of an *Image of Thought* is developed precisely to describe a notion of what it is to think *and* a practice of thinking and his notion accounts for the inextricable interwovenness of a practice and a notion of what it is to think (Deleuze 1997, 169 f.). Because I believe it is crucial to explore *Mathematics* as both a notion *and* a practice of thinking I draw from Deleuze to understand *Mathematics*. My variation of speaking of an *Image of Thinking* rather than an *Image of Thought* gives special attention and emphasis to thinking as a practice because the core focus of my work is to understand what kinds of epistemic practices and operations originate from epistemic notions and values.

My understanding of *Mathematics* as an Image of Thinking is interwoven with an understanding of *Western*[1] thought that diverges from employing 'Western' as a primarily geographical term. I draw from Andrea Nye (1990), Bonnie Shulman (1996), and Sara N. Hottinger (2016) in employing the term *Western* in a political sense in which *Western* refers to a historically embedded entitlement to and upkeep of power. Following Bonnie Shulman (1996), Denise Ferreira da Silva (2017), and Sara N. Hottinger (2016), I understand a highly distinct and yet deeply universalized understanding of *Mathematics* to be the product of Western thought and Western maintenance of power. Hence, the capitalized and italicized writing of the word 'mathematics' emphasizes that a) I am referring to a specific understanding of mathematical notions and practices, and b) that this understanding of what it is to 'think mathematically' is deeply universalized as it is alleged to be the *only way of engaging in mathematical practice*.

The practice of universalization is not an erasure of *wild mathematics* nor of epistemic wildness, but rather a constant practice of attacking the thinking-being in non-accordance with *Mathematics*-centric scripts in an attempt to erase them.

1 I choose to capitalize 'Western' because it aligns with my capitalization of the universalized Western mathematical practice as '*Mathematics*'. In both cases, the capitalization symbolizes the universalization built into these very notions, which serves to then remind us – just upon reading the word – how these are in fact distinct modes and concepts that are made to appear universal.

I explore how *Mathematics* is thereby formed through a particular and distinctly Western and patriarchal manner of understanding *mathematical truth* and *mathematical proving*. I explore how both the notion of *truth* and the notion of *proving* draw from an axiomatic method, which identifies itself through the aim to establish *truth beyond doubt* by deducing further knowledge from propositions understood as epistemically certain (*axioms*) (cf. Shulman 1996, 429). This first chapter explores the axiomatic as an Image of Thinking itself to explore its relation to *Mathematics* and its historical and political context and texture.

My study of *Mathematics* as an Image of Thinking begins in contemporary scholarship on *Mathematics* and moves from there into a historical study. I begin with contemporary works, because this thesis originates from the question of *Mathematics* as an implicit reference point in contemporary Western politics.

I employ the notion of politics in a deliberately wide-ranging manner that draws from Isabelle Stengers usage of the notion in which political is that which is concerned with authority (Stengers 2007). I favour this term over 'societies or 'culture' as it does not evoke any imagery of closed-off systems but rather focuses on a dynamic or a movement of the makings and unmakings of authority.

For my historical study I choose two periods in the history of philosophy that are – implicitly as well as explicitly – referenced in contemporary understandings of *Mathematics*: Ancient Greek philosophy and the philosophy of Enlightenment.

Ancient Greek philosophy is widely recognized as the place of origin of the axiomatic method (cf. Shulman 1996, Nye 1990), which makes it a crucial period to study as part of a core part of this chapter's endeavor tore-trace the relationship of *the axiomatic method* and *Mathematics*.

The philosophy of the Enlightenment is often referred to as crucial reference point for *Modern Science* and the expansion of *Rationality* (e.g. Pinker 2018). Furthermore, as I will show through my study of contemporary scholarship, the period of Enlightenment often forms a reference point for contemporary self-understandings of Western cultures (see ibid.). Therefore, the period of Enlightenment is the second historical study I choose to conduct to analyze the relationship of *Mathematics* and *Rationality*.

My contemporary study explores scholarship from different disciplines to establish that *Mathematics* has an existence that exceeds disciplinary boundaries because it moves through them all in its own way. My two historical studies then focus on crucial philosophical work of the periods of Ancient Greece and the Enlightenment. I focus on philosophical works because in both periods philosophy played a distinct role in shaping and showing the Zeitgeist of its period, which makes crucial philosophical works of each period relevant and suitable representatives and thus exemplary case studies of the respective Zeitgeist and the Zeitgeist's relation to *Mathematics.* Furthermore, I focus on philosophical works, because researching the makings of *Mathematics* through philosophical work helps to explore the relationship of *Mathematics* and *Philosophy* – a task I consider to be part of understanding *Mathematics* as an Image of Thinking.

In my study of contemporary works on *Mathematics*, I consider the books of psychologist Steven Pinker, physicist Michael Brooks, and mathematician Jordan S. Ellenberg. Their best-selling books mirror various disciplines and therefore reflect the wide-ranging appeal and acceptance of the understanding of *Mathematics* they contain. In examining these works I focus on the epistemic characteristics and practices they associate with *Mathematics* and on how these characteristics and *Mathematics* extend beyond *Mathematics* as an academic discipline. In particular, I focus on the relationship of *Mathematics* with bodies of Western self-understanding and politics as it can be traced in these books.

In my study of Ancient Greek philosophy, I draw from the philosophies of Parmenides, Plato and Aristotle. Using exemplary text passages, I establish the core characteristics of the concepts of *truth* in the respective philosophies and analyze the relationship these notions of *truth* have towards *Mathematics*. As this chapter leaves no room for discussion and interpretation of these philosophies, I only draw from those aspects that can be proven both in primary literature itself and in various supporting works of secondary literature.

In my study of the philosophy of Enlightenment, I turn to René Descartes and Immanuel Kant. As both are prominent figures in Enlightenment philosophy itself as well as in its contemporary reception today (cf. Pinker 2018, 4), I choose to examine them in this historical retracing that moves from a point of contemporary political concern to retrace the intellectual histories that got us here – that got us this *Mathematics* and this *Rationality*. Here, too, I work closely with primary literature of the two philosophers as well as secondary literature whose analysis supports the primary sources and is well-received. Resultingly, I focus on the question of how *Mathematics* is related to *Rationality*.

The Deleuzian notion of an *Image of Thinking* guides these studies: An Image of Thinking describes a relational field of epistemic values, operations, and practices. And this is precisely what I research here. As I research *Mathematics* as an Image of Thinking, I aim to establish what characterizes its existence as an Image of Thinking and to explore the relationship of *Mathematics* to other Images of Thinking such as *Rationality* or to epistemic ideals such as *truth*.

1.2 The Politics of *Mathematics*
Violence and *Nonviolence* in Contemporary Times

This first section of my historical reconstruction of *Mathematics* as an Image of Thinking focuses on contemporary works and attitudes towards *Mathematics*. In doing so, it traces the *Mathematics-Rationality-Human* Continuum to account for a) the context and construction of the continuum and to b) investigate the images of thinking established by the continuum. Here I focus on showing how *Mathematics* exists as an image of thinking that is anchored in the consciousness of Western society as a whole in which *Mathematics* forms the notion of *common sense,* what is understood as *Mathematical thinking*, and also what is construed as *thinking* more generally. In my analysis grounded in contemporary times I choose to examine three non-philosophical works. This allows me to first establish *Mathematics* as an Image of Thinking exceeding *Mathematics* as an academic discipline by showing the deep-rooted reality of the *Mathematics-Rationality-Human* Continuum, which forms worlds in which *Mathematics* moves as a largely unspoken conceptual point of reference for *the Rational* and *the Human*. All three chosen works share a broad popularity in scientific as well as extra-scientific spaces and are exemplary cases of the *Mathematics-Rationality-Human* Continuum at work for they showcase the continuous making of a conceptual relationship between *the Mathematical* and *the Human* that is mediated through the notion of *the Rational*.

I begin my analysis with the book *Enlightenment Now – The Case for Reason, Science, Humanism and Progress* by Steven Pinker (2018). I draw on this work to show the conceptual interwovenness of the notions of *reason, humanity* and *progress*. All three of these notions are strikingly prominent in Pinker's work and in his understanding of *the Enlightenment* as a distinct form of intellectuality. I will show how this concept constructs a *reason-human* continuum and attributes the label of *progress* to this continuum and resultingly is an endeavor of world-making, which always implicitly references and centers *Mathematics* as a mode and notion of thinking – and eventually, a notion of being too.

I will then turn to two works that are more explicitly about *Mathematics*. I draw from *How Not to Be Wrong – The Power of Mathematical Thinking* by Jordan Ellenberg (2014) to further examine the interwovenness of *Mathematics* with the notions of *reason* and *thinking*. I draw from *The Art of More – How Mathematics Created Civilization* by Michael Brooks (2021) to further analyze the conceptual interwovenness of *Mathematics* and Western modes of self-understanding, which are represented through the notions of *civilization* and *progress*.

So, let me begin this examination with Pinker to establish the intellectual landscape that fashions the *Mathematics-Rationality-Human* Continuum. *Enlightenment Now* refers, as the title suggests, to epistemic and political ideals, which Pinker understands as 'Enlightenment ideals.' His understanding of the Enlightenment is primarily based on Immanuel Kant's statement on 'the liberation from self-inflicted immaturity' (cf. Pinker 2018, 7). He understands Enlightenment ideals to help in 'understanding the human condition' and advocates for employing the 'ideals of the Enlightenment' to contemporary times and politics (Pinker 2018, 5, 6). The notion of a distinct 'human condition' plays a central role for Pinker in the very scope and conceptual make up of his endeavor; therefore, establishing early on that the notion of *the human* is central to Pinker and his understanding of the other core concepts of his work such as *Rationality*, *progress*, and *reason* (Pinker 2018, 5, 6).

Pinker associates the era and philosophy of the Enlightenment with a distinct hope for political as well as day-to-day contexts writing that 'some good' will come from recalling 'the ideals of the Enlightenment'.: This is one of many instances, where Pinker constructs an epistemic goodness he believes to be an ahistorical creature, which is 'good' precisely because of its alleged ahistoricity and impartiality (Pinker 2018, 349–350). He goes on to establish the three notions he understands as 'Enlightenment ideals': *reason, science, and humanism* (Pinker 2018, 349–350). Accordingly, the notion of 'goodness' that Pinker puts forth is characterized through the notions of *reason*, *science*, and *humanism*. This is crucial because it shows how Pinker moves at an intersection of epistemic and normative questions and ideals.: The 'good' he envisions is realized through epistemic values, beliefs, and practices (Pinker 2018, 349–350). For Pinker the normative and the epistemic form a continuum too in the sense that according to Pinker there are good epistemic practices and bad epistemic practices in which 'good' and 'bad' are moral as well as epistemic categories for Pinker and in which they are distinctly constructed as ahistorical (Pinker 2018, 349–350). This continuum of the normative and the epistemic will become more apparent as we read Pinker in his own words.

For now, let me say that I understand this conflation of the normative and the epistemic woven through with a concept of ahistoricity as a decisive aspect of the conceptual landscape of the *Mathematics-Rationality-Human* Continuum. The continuum is normative in that it fashions norms, normativity, and normalcy for images and modes of thinking and being.

Pinker characterizes *reason* as follows:

> Opposing reason is, by definition, unreasonable. But that hasn't stopped a slew of irrationalists from favoring the heart over the head, the limbic system over the cortex, blinking over thinking, McCoy over Spock.
> *Enlightenment Now (2018), 351*

In this excerpt, it becomes clear how universally and binarily Pinker thinks of the concept of *reason*. To Pinker, *Rationality* and *Irrationality* exist, with the former being the hero and the latter the antagonist. He bases the *universality* of this *goodness* of *Rationality* in defining the *other* to *rationality* as the *irrational*. This reference thereby expresses a claim to epistemic authority for notion that *thinking* is considered *rational*. The reference to the definition of *Rationality* is thereby the consolidation of what Deleuze calls *common sense*: the existing understanding of what is *credible thinking* and *knowledge* in which what is not is set as its indisputable basis (Deleuze 1992, 171 f.). The moral dimension of this binary distinction is treated as a necessary one, which is not understood as worth discussing as the sense that *Rationality is the higher value* over *Irrationality* seems to be a universal and necessary one for Pinker. However, this view is more than a judgment, which is made and culturally shaped; he ascribes to it an ahistorical character.

Following this, he names a series of judgments that he understands as exemplary of decision-making or of *Irrationality* over *Rationality*, in which he portrays the idea of *body* and *mind* as opposite, likewise hierarchized modes into the same binary that defines his *Rationality* and *Irrationality* in which the sphere of the *corporeal* is conceptually linked to the idea of *Irrationality* and the idea of the *mind* to the idea of *Rationality*.

The relationship in which Pinker places the idea of *Rationality* to the sphere of the political is also a fundamental aspect of his concept of *reason* and is exemplified in the following excerpt in which Pinker refers to a study where participants were categorized according to their political orientation and asked to assess the *truthfulness* of individual *news reports:*

> If the left and the right are equally stupid in quizzes and experiments, we might expect them to be equally off the mark in making sense of the world. [...] I've been arguing that the main drivers [of human progress] were the nonpolitical ideals of reason, science, and humanism, which led people to seek and apply knowledge that enhanced human flourishing.
> *Enlightenment Now (2018), 362–363*

Pinker uses this study to conjure up the image of *reason* that is clouded and misled by *politics*[2], He positions political attitudes and beliefs to appear as sources of the transfiguration of this *reason and* thus positions *reason* as *outside the political* –as a way of thinking without an inherent political dimension. The first headline which he quotes as a summary of the study refers explicitly to *Mathematics* and reads the results of the study as proof that *politics* is demolishing "our" ability to *think mathematically.* This title is based on an understanding of *Mathematics* that renders *mathematical thinking* as maximally *apolitical thinking* and thus evokes particular potential for indignation in relation to its linking with *politics* in the title. The second title cited works in a similar way, describing thinking that is guided by political conviction as *stupid*. Certainly, Pinker cites both titles as the exaggerations they clearly are; however, they also tellingly applies the same morally dimensioned binary of *reason* and *politics.*

This moral binary becomes particularly clear in the second paragraph of the quote in which Pinker chooses thinking against the background of political convictions as the antagonist by specifically marking it as *stupid,* thus explicitly banishing it from the concept of *reason*. The heroes are *non-political ideals* of *reason*, *science,* and *humanism*. In declaring these ideals as *non-political,* Pinker ignores the political dimension and history inherent in all these concepts. He thus declares the political implications of *reason*, *science,* and *humanism* to be *apolitical* and given thereby granting them unprecedented epistemic authority. This authority has a direct moral dimension: being on *the good side* means granting this epistemic authority. The idea of reason as apolitical or politically neutral is one that Pinker repeatedly puts forward:

> [...] It's in the very nature of argument that people stake a claim to being right. As soon as they do, they have committed themselves to reason [...].
> *Enlightenment Now (2018), 351–352*

2 Pinker uses the term "politics" to refer to political convictions as well as a general view of the political dimension of narratives, events, etc.

Here Pinker universalizes and naturalizes striving for epistemic authority – through the act of wanting to *be right* – as an inherent part of all human argumentation and debate.

> As demonstrated by the following excerpts, the moral dimension of his concept of knowledge and reason is essentially fed by the connection between these ideas and his concept of humanism. Progress consists of deploying knowledge to allow all of humankind to flourish in the same way that each of us seeks to flourish.
> The goal of maximizing human flourishing [...] may be called humanism. [...] Knowledge of the world is derived by observation, experimentation, and rational analysis. Humanists find that science is the best method for determining this knowledge [...].
> *Enlightenment Now (2018), 410–411*

Pinker describes *humanism* here as a specific way of producing and applying *knowledge*, which assumes universal aspirations of "human flourishing". Resultingly, he deepens the political and moral dimension of his concepts of *knowledge* and *reason* by construing them as the means of fulfilling *universal human needs.*

The construction of *epistemic goodness* that goes hand in hand with this conceptual interweaving becomes clear in the following quote:

> The same is true of the common argument that the claims of science are untrustworthy because the scientists of some earlier period were motivated by the prejudices and chauvinisms of the day. When they were, they were doing bad science, and it's only the better science of later periods that allows us, today, to identify their errors.
> *Enlightenment Now (2018), 391*

Pinker assumes that *bad science* as an epistemic and moral category exists ahistorically and universally. In doing so he overlooks the fact that *epistemic and scientific goodness* are concepts that are themselves subject to processes of change and interpretation and are inherently political in that they negotiate questions of the authority of *knowledge* and expression.

Finally, I draw on an excerpt from Pinker's work that shows how his understanding of *knowledge* as *politically neutral* affects his understanding of violence:

> *Isn't internet trolling a form of violence? Isn't strip-mining a form of violence? Isn't inequality a form of violence? Isn't pollution a form of violence? Isn't poverty a form of violence? […]*
> As wonderful as metaphor is a rhetorical device, it is a poor way to assess the state of humanity. […] It [moral reasoning] also requires distinguishing rhetoric from reality. […] Finally, improving the world requires an understanding of cause and effect. Though primitive moral institutions tend to lump bad things together and find a villain to blame them on, there is no coherent phenomenon of "bad things" […].
> *Enlightenment Now (2018), 47 [Italics not my own]*

Here Pinker establishes that he understands different concepts of violence as semantic gimmicks or metaphors thus dismissing a) the existence of various forms of violence and b) the interwovenness of different forms of violence with one another. Furthermore, he strongly advocates for a separation between different situations of precarity.

I advocate for understanding violence as a continuum, through which various forms of violence make one another happen. In other words: one form of violence is never to be separated from another in my mind. Understanding violence as a continuum means troubling accounts of violence that cling to the logic of *separateness*[3] like Pinker's does.

Pinker constructs the idea of different forms of violence as inherently *irrational* thus rendering it 'undiscussable' in the context of his thinking: As a result, the only understanding of violence that can be 'credible' to Pinker one that aligns with Pinker's understanding of 'ahistorical-apolitical knowledge', which according to his metric is an understanding of violence construed through the logic of separateness. It This understanding of violence through a logic of separation is a consequence of how Pinker understands knowledge, for, if he were to admit different concepts of violence, he would have to consider epistemic violence as well as violence that historically situates 'goodness' and 'badness', but his very worldview depends on a normative-epistemic understanding of violence that claims to be an ahistorical creature. Thus, we can summarize Pinker's

3 In alignment with the work of da Silva (da Silva 2016) and Erin Manning (Manning 2020) I understand *separateness* and *separability* to be concepts interwoven with whiteness and – in my understanding – with masculinity too. *Separability* is how both whiteness and masculinity come into existence because the notion is part of their fabric as concepts and practices.

image of (Mathematical) thinking through the following claims characteristic of his work.

1) Pinker constructs 'credible knowledge' in relation to *Mathematics* as an image of thinking by establishing the knowledge-creation based in *Mathematics* as form of logic – e.g. *data* and the *sciences* – are forms of knowledge-creation that are *rational* and *credible*. This construction is exactly what I draw attention to when employing the notion of the *Mathematics-Rationality-Human* Continuum in which *The Rational* as an Image of Thinking only exists in relation to *the Mathematical* as an Image of Thinking and *the Rational* as an Image of Thinking is interwoven with *the Human* as an image of thinking-being. This too shows how thinking and being build a continuum making *the Mathematical*, *the Rational*, and *the Human* into scripts for thinking-being.
2) Pinker constructs *reason, knowledge, progress* and *humanism* as universal and ahistorical notions. Not only does this understanding distinguish these four notions, but it also serves to ground the normative superiority Pinker attributes them. They are the ideals he advocates for precisely because he inhabits an intellectual world-making that believes these four notions to be 'good' in an ahistorical sense of the word 'good.' This world-making is the world-making of the *Mathematics-Rationality-Human* continuum in which *Mathematics* builds a continuum with *reason, knowledge, progress,* and *humanism* by representing a form of proof of the universalism and ahistoric epistemic certainty that *reason, knowledge, progress,* and *humanism* strive for – according to Pinker.
3) The notion of *reason* is inherently 'good' and desirable in Pinker's thought. R*eason* is constructed as a) an ideal for *thinking* and b) as the thing one partakes in through every endeavor of thought. This second aspect is especially crucial because it establishes that thinking and being in fact build a continuum as in Pinker's world-making 'human thinking' is defined through *reason*. So, *reason* becomes a signifier for *the human* because it signifies the thinking that makes a creature *human*.
4) Pinker presupposes an ahistorical 'epistemic goodness' and 'epistemic badness,' while is simultaneously always a moral goodness and badness too. *Mathematics* is woven into this construction of epistemic goodness and into the construction of its ahistoricity. As a result, it is knowledge that references *Mathematics* as an Image of Thinking – like numbers or calculations –

that Pinker construes as 'epistemically good.' Mathematics thus authorizes Pinker's notion of 'epistemic goodness.'

I now examine the work of mathematician Jordan Ellenberg to move my examination of the *Mathematics-Rationality-Human* continuum more explicitly to a) an analysis of *Mathematics* as an Image of Thinking and b) to engage a conception of *Mathematics* as it is referenced in more general notions of *thinking*.

In his book, Ellenberg sets himself the goal of providing an introduction to *Mathematical thinking* which also simultaneously advocates for *Mathematical thinking* as a particularly reliable way of thinking, i.e., a way of thinking through which one '*is not wrong*.' In doing so, he explicitly interweaves *Mathematics* with more general understandings of *thinking* and *knowledge*. This serves to which showcase and reproduce the *Mathematics-Thinking* continuum built into the *Mathematics-Rationality-Human* continuum. He begins his book as follows:

> Math is woven into the way we reason. And math makes you better at things. Knowing mathematics is like wearing a pair of X-ray specs that reveal hidden structures underneath the messy and chaotic surface of the world. Math is a science of not being wrong about things [...]. With the tools of mathematics in hand, you can understand the world in a deeper, sounder, and more meaningful way.
> *How Not To Be Wrong (2015)*, 2

In this excerpt, Ellenberg describes *Mathematics* as a way of thinking that does not only take place in those spheres that are commonly understood as the realm of *Mathematics*; rather, as a way of thinking that is fundamentally inscribed in thinking and reasoning. For him, *Mathematics* is in this sense a more developed form of fundamental *Human Rationality, which according* to Ellenberg, makes one fundamentally *better at thinking* because it is a thinking technique of '*not being wrong*.' This formulation assigns epistemic authority to *Mathematics* and the thinking that relies on it. Furthermore, it highlights epistemic dominance as an underlying goal for Ellenberg, as this concern of thinking is not made explicit or justified as such, but rather implied as a matter of course – as if all thinking were such that has epistemic dominance ('not being wrong') as its goal.

Ellenberg goes on to explain his understanding of the relevance of *Mathematics* along the lines of the concept of *common sense:*

> Mathematics is common sense. On some basic level, this is clear. How can you explain to someone why adding seven things to five things yields the same result as adding five things to seven? You can't: that fact is baked into our way of thinking about combining things together.
> *How Not To Be Wrong* (2015), 10–11

Ellenberg describes *Mathematics* here as *common sense in that* he understands *Mathematical Thinking* as that thinking which is precisely that of *common sense*. He thus conceptualizes the concept of *Mathematics* along the lines of what is generally considered *good, sensible thinking* and vice versa. In doing so, he universalizes and idealizes *Mathematics* and constructs *Mathematics* and *common sense* in mutual dependence thus organizing his concept through constructing a relationship of mutual authorization, which interweaves the two images of thought.

In this excerpt, he also raises the figure of inexplicability. According to Ellenberg, *Mathematical Thinking* is inexplicable because it shows the *most fundamental truths*. Like Pinker, he applies the same idea to *thinking* and *logic*.

On the following pages, Ellenberg describes the relationship between *Mathematics* and *Common Sense* in more detail:

> Mathematics is the study of things that come out a certain way because there is no other way they could possibly be. [...]
> Math is like an atomic-powered prothesis that you attach to your common sense, vastly multiplying its reach and strength. [...] I find it helpful to keep in mind an image of Iron man punching a hole through a brick wall. [...]
> Without the rigorous structure that math provides, common sense can lead you astray.
> *How Not To Be Wrong* (2015), 12–13

He begins his description, by reintroducing the figure of *mathematical knowledge* as the *necessary knowledge* – the *knowledge* that cannot be otherwise. He thus by definition places *mathematical knowledge* in a position of special epistemic authority.

Following this, he describes *Mathematics* as an extension and elevation of *common-sense thinking – normal, logical thinking* – and describes *Mathematics* as an epistemic hero, similar to Pinker's description of *reason*. This element of heroization is reinforced by the reference to *Iron Man* which also underpins the moral dimension observable in Pinker. As in most superhero films, there is no serious debate about which *side* is *right.* Ellenberg also assumes a universal *epistemic goodness* and uses both *Mathematics* and the idea of *common sense* as examples of this epistemic goodness whereby neither this moral binary itself – nor the assignment of *Mathematics* and *common sense* is explicitly up for discussion.

Ellenberg expands on the idea of *common sense* with reference to Marquise de Condorect when he writes explicitly about the relationship between *truth* and *majority:*

> If the majority of people believe something, Condorect said, that must be taken as strong evidence that it is correct. We were mathematically justified in trusting a sufficiently large majority [...]. 'I must act not by what I think reasonable' Condorect wrote, 'but by what all who, like me, have abstracted from their own opinion must regard as conforming to reason and truth.'
> *How Not to Be Wrong (2015), 388*

Ellenberg argues that majority is thus an indication of *truth* and refers to *Mathematics* as an authorization of this argument by describing this thesis as *mathematically justified.* However, he does not explain what this *mathematical justification* actually consists of and why it should be credible. Instead, he ends this section by quoting Condorcet's call for trust in majority opinions. The idea that the majority can secure *truth* implies a fixed concept of truth that is not a matter of negotiation and overlooks the fact that positions capable of winning a majority are often shaped by historically and culturally specific *common-sense knowledge*. This section thus also shows that Ellenberg understands common sense knowledge not as situated, but as *neutral, ahistorical knowledge.*

The fact that for Ellenberg *Mathematics* is not an accumulation of certain facts, but actually a way of thinking that symbolically stands for the ideal of absolute epistemic certainty and its fulfillment becomes clear in the following excerpt:

> They [mathematical ideas] are the go-to tools on the utility belt, and used properly they will help you not to be wrong.
> *How Not To Be Wrong (2015), 16*

Here Ellenberg explicitly describes how *Mathematical Thinking* goes beyond what is conventionally understood as *mathematical.* He understands *mathematical thinking* as that thinking which establishes complete *epistemic certainty* in which he describes *Mathematics* almost as a place of certainty –is possibly not only *epistemic certainty,* but also a fundamental sense of definiteness and constancy. He therefore understands *mathematical thinking* as tools of thought that are designed *not to be wrong* – that is, to have epistemic authority.

The last sentence of the section paints an image of *Mathematics* as a sphere that is, as it were, far removed from the world – or at least a world of *disorder* and *indeterminacy*. Here, too, an understanding of *(Mathematical) Truth* that is universal and ahistorical is it is clearly implied. and defines *Mathematics* as the epistemic sphere that is politically and culturally untouchable and untouchable.

Similar to Pinker, Ellenberg's evaluation along a binary of *certain* and *uncertain knowledge is* a fundamental element of his work, which is expressed particularly explicitly in the following quote:

> People usually think of mathematics as the realm of certainty and absolute truth. In some ways that's right. We traffic in necessary facts: $2 + 3 = 5$ and all that.
> But mathematics is also a means by which we can reason about the uncertain, taming if not altogether domesticating it.
> *How Not To Be Wrong (2015), 425*

Here Ellenberg construes *Mathematical knowledge* as 'certain knowledge' and *Mathematics* as the way of thinking that 'domesticates' spheres of uncertainty – thus making uncertainty into a creature to be tamed and *Mathematics* into a technique of taming. There is a hierarchized binary of 'certain versus uncertain knowledge' implied here: 'uncertain knowledge' is the knowledge to be tamed by 'certain knowledge.' This implies a similar sense of normativity as I have found in Pinker in which there is a 'good knowledge' and a 'bad knowledge', and it is the notion of certainty that separates the two.

Ellenberg's *image of Mathematical thinking* can be summarized through the following core claims:

1) Ellenberg understands *Mathematics* as a way of thinking that leads to particularly strong 'epistemic certainty.' Thus, he conceptualizes *Mathematics* through a notion of epistemic certainty much like Pinker did. Ellenberg understands thinking as a striving for epistemic certainty and *Mathematics* as a mode of thinking, which is particularly strong at creating this 'epistemic certainty.': With this claim, By construing *Mathematics* as the way of thinking that fulfills what all other thinking strives to achieve, he conceptu-

alizes *Mathematics* as a reference point for thinking more generally. Indeed, this is the *Mathematics-Thinking* continuum at play through which thinking itself comes into existence with only an implied reference to *Mathematics*.

2) He understands *Mathematics* and 'common sense' in relation to each other. *Mathematics* is the extension of 'common-sense thinking.' This further establishes and strengthens the *Mathematics-Thinking* continuum moving through Ellenberg's work as it moves through Pinker's work: *thinking* is *thinking* in relation to *Mathematical thinking*.
3) Ellenberg identifies the notions of *majority* and *normality* with *truth* by establishing the *majority* and *normality* as indicators of *truth*. His notion of *normality* here is resultingly conflated with *majority* and both are conceptualized as reliable indicators for *truth*. This construction renders any understanding of violence or exclusion built into notions of *normality* non-credible.
4) *Mathematics* symbolizes 'secure knowledge' and even more so '*epistemic goodness*,' which is understood as a universal value. This further authorizes the *Mathematics-Thinking* continuum in its normative dimension.

My final case study for the study of *Mathematics* as a contemporary image of thinking-being is the book *The Art of More* written by physicist Michael Brooks (2021). Similar to Ellenberg, his aim is to make *Mathematics* accessible and to arouse fascination for it. His concrete strategy consists of a kind of retelling of (Western) world history with *Mathematics* as the hero.

Brooks begins his book with a localization of *Mathematics* in "our culture," whereby he focuses exclusively on Western culture, without explicitly naming it:

> […] you learned to count because of cultural pressures. Those pressures came from an interesting place: a deeply ingrained cultural wisdom that tells us that mathematics matters.
> *The Art of More (2021), 2*

Brooks paints an image of a *deep-seated cultural wisdom* about the meaningfulness of *Mathematics*, whereby he understands meaningfulness as an intrinsic element of *Mathematics*, rather than as a subject of culturally specific negotiation and the scene of discourses on the politics of meaning. He goes on to write:

> […] mathematics *is* supernatural, in that we have used it to go beyond the natural.
> *The Art of More (2021), 2*

This section shows the element of mystification in the image Brooks draws of *Mathematics and* also shows similarities to Ellenberg's superhero metaphor. Positioning *Mathematics* as *supernatural* in terms of its epistemic practice and potentiality is therefore an authorizing mechanism of authorization, which functions – quite literally – by conceptually placing *Mathematics* above *normal, natural thinking*.

Brooks combines the systematic idealization of *Mathematics* with the idea of *being human*:

> I learned none of this at school. […] I never learned what mathematics had done for us as a species or how we came to invent it.
> *The Art of More (2021), 4*

Here Brooks already hints at what he wants to show in the course of the book. Firstly, that he starts from *Mathematics* as a special achievement of *human thought* and that secondly he ascribes it a special relevance in the history of 'man' as a whole species. The connection between *Mathematics* and *human history* authorizes *Mathematics* and positions it as a respectable, even honorable entity and establishes a strong conceptual proximity between *Mathematics* and 'man'. The concept of the *human species* is already an exclusive one, insofar as Brooks writes about a specifically Western *Mathematics* without naming it.

The chapters of the book are based on this very image of *Mathematics* as the *hero of history*. Brooks tells individual stories, which he understands as "challenges for humanity" (here, too, he moves almost exclusively in the context of Western culture) while describing a sub-area of *Mathematics* as salvation. He does so in order to then explain this sub-area of mathematical discipline in its basic features. This pattern can be exemplified by his summary of HIV and AIDS:

> Today, just over a decade later, HIV infection is no longer a death sentence. In fact, people with HIV live relatively normal lives. What happened? Calculus.
> *The Art of More (2021), 128*

At this point, I will ignore the fact that this way of narrating historical events is largely under-complex and in some cases, highly misleading. Instead, I will focus on how this narrative pattern conceptualizes *Mathematics. Mathematics* appears as a hero and is thus, on the one hand, assigned the part of a specifically masculine trope[4] and thus constructed as an aspect of *masculinity* in the binary of *masculinity* and *femininity*. In addition, this narrative deepens the authorization of *Mathematics* as a special epistemic achievement, which in a sense plays "in a different league" than other epistemic practices.

There is also a link between the idea of a *modern world* and the idea of *Mathematics* that runs through Brooks' work and comes to a head with the following claim:

> […] this strange mathematical creature [imaginary numbers] is real enough to power almost everything in the modern world.
> *The Art of More (2021), 178*

Brooks uses the Western-influenced concept of modernity frequently throughout his book – always without reflecting on the cultural and historical context of the term. For Brooks, the *modern world* appears as a consistently positive figure that is associated with the idea of progress and universalized as a value in itself. The idea of the 'modern world' and that of *Mathematics* mutually authorize each other.

In the following excerpt, the aspects of Brooks' image of *Mathematical thinking* that have been emphasized so far appear grouped together:

> Developing math allowed us to dissect and dismantle nature's patterns and symmetries and, like gods, recast them in ways that serve our interest. […] eventually we found ourselves establishing civilizations. […] We call these calculus, and they enable us to realize a range of human aspirations […].
> […] the human story itself is inextricably interwoven with mathematics. Columbus' journey to the Americas relied on understanding the properties of triangles […]. Mathematics provides the sculptor's chisel that shaped the Renaissance and the ammunition that has engendered centuries of military

4 Sara Hottinger discusses this heroization of *mathematicians* as a *masculine* and *Western* construction: according to Hottinger, it creates "a specific subjectivity – highly rational, alienated from humanity, a hero who shapes Western culture." For more details on the power-critical consideration of the heroization of mathematicians, see Hottinger 2016, 89–125.

> success.
> *The Art of More (2021), 3–4*

Mathematics appears here explicitly in the role of the dominant designer. It is described as the basis for taming nature and reshaping it according to one's own ideas. Following on from this image, Brooks links *Mathematics with* the concept of *civilization,* which is also not illuminated in its colonial historicity and significance. He conceptualizes *Mathematics* and *civilization* already here in a direct connection and also creates a relationship of mutual authorization between the two concepts as well as the universalization of underlying values. *Civilization* appears as a value in itself and *Mathematics* is understood as valuable insofar as it founds *civilization, which* is *civilization* understood as valuable insofar as it founds *Mathematics.*

Similar to Pinker, Brooks also applies a universalized concept of *human aspirations*, which is based on inherently human interests and works with an exclusive and Western-centric concept of the *human being*. *The* same can be said of his concept of *human history*. He presupposes that there is one *human history,* while he actually narrates Western-centered history. This exclusive and colonially influenced concept of the *human being* is here directly interwoven with *Mathematics* in that he understands *Mathematics* as a structural element of the supposedly universal *history of mankind* and the fulfillment of supposed universal *human ambitions*. In doing so, he conceptualizes *humanness* specifically with reference to *Mathematics*, and in doing so, constructs both concepts as Western and masculine.

His linking of *Mathematics* and *civilization* is a central part of his summary at the end of the book:

> Scholars rarely agree on what exactly defines a civilization [...].
> [...] This should, in fact, be the first – perhaps the only – requirement. I'm talking about mathematics, of course. [...]
> *The Art of More (2021), 285–286*

Brooks proposes here to declare *Mathematics as the* primary or even sole defining characteristic for the concept of *civilization*; thereby, strengthening the conceptual link between *Mathematics* and *civilization*, which in turn is linked to the idea of *being human*.

> Mathematics has shaped the very experience of what it means to be human and left its mark on all of us [...]. So [...] perhaps we can now all agree on something: that mathematics created all of us.
> *The Art of More (2021), 291*

Brooks here establishes his understanding of *Mathematics* as signifying and characterizing what it is *to be human*. He further showcases the *Mathematics-Human* continuum as it moves through his work through his use of an exclusive 'us,' that too is signified by *Mathematics*. There is a 'human we' moving through Brook's work that is always implicitly Western and defined by a decisive relationship to *Mathematics*. Brooks employs *Mathematics* here as an idea to form this exclusive 'us' and to universalize and thus authorize it. He ends his book by officially defining *Mathematics* as the one signifier – the identifying element of the notions of *humanity* and *civilization*. This further establishes the *Mathematics-Human* continuum at the core of his work and the notion of civilization therein points towards the colonial exclusivity of this Human that is the *Human* of the *Mathematics-Human* continuum.

Brooks' *image of (Mathematical) Thinking* can be summarized as follows:

1) Brooks understands *Mathematics* as an instance of the realization of universalized 'human aspirations' and conceptually links *Mathematics* to Western history and an exclusive concept of *being human*. This showcases the *Mathematics-Rationality-Human* continuum and hints at the exclusivity of the thinking-being legitimized through and within the continuum.
2) Brooks conceptually interweaves *Mathematics* with *civilization* and *modernity* through relationships of mutual authorization and mutual legitimacy. This once more allows us to understand the normative quality of the *Mathematics-Rationality-Human* continuum as establishing *Mathematics* as a thinking-being with distinct epistemic authority.

The works of Pinker, Ellenberg, and Brooks all partake in the making of a contemporary *Mathematics* that is characterized through the following criteria:

1) *Mathematics* is understood to form knowledge with a distinct and special epistemic authority.
2) The special epistemic authority attributed to *Mathematics* bears a moral dimension of 'epistemic goodness'.

3) *Mathematics* is conceptually interwoven with more general notions of thinking, 'common sense,' and of *being human*.

1.3 Inventing *Mathematics*
The Axiomatic Method in Ancient Greek

Now that we have witnessed three instances of the *Mathematics-Rationality-Human* continuum at work, I seek to trace some of the historical makings of both *Mathematics* and the *Mathematics-Rationality-Human* continuum. *Mathematics* as an image of thinking is fundamentally grounded in the axiomatic method that draws from Ancient Greek Philosophy (Shulman 1996). The *Mathematics-Rationality* continuum is often attributed to the Enlightenment and the philosophy of this period. Therefore, my next two destinations in the study of *Mathematics* are the philosophy of Ancient Greece and the philosophy of the Enlightenment.

As the man who first developed the image of thinking that was later formed into the axiomatic method, Parmenides is generally regarded as the "father of logic" (cf. Steuben 1981, 94). The axiomatic method thus became the heart of *Mathematics* and *Rationality*[5] as images of thinking (cf. Shulman 1997, 429). Only a fragmentary didactic poem of his philosophy has survived, which itself does not mention *Mathematics*, but is strongly associated with *Mathematics* as 'a method of thinking' (cf. Steuben 1981, 67).

This poem narrates a story in which Parmenides is the human protagonist introduced to the art of recognizing *truth* by various goddesses (cf. Shulman 1997, 430). This 'world of truth' has the character of an irrefutable system that is explicitly separated from 'the sphere of error,' which is 'the common sphere of mortals' (cf. Steuben 1981, 37).

Various aspects and scenes of the didactic poem are still the subject of polarizing debate today (cf. e.g. Steuben 1981, 115 or Nye 1990, 9). These include, above all, the question of whether Parmenides' path to truth has the character of a revelation or that of a personal intellectual endeavor (cf. e.g. Steuben 1981, 86). However, there is agreement that Parmenides describes a realm of infinite *truth* that is not actually accessible to humans, but to which there exists a

5 I use italics to clarify the character of a conception: *mathematics*, for example, is a specific epistemic structure that is subsumed under the term *mathematics* - not all mathematical understandings or conceptions.

path that humans can take –the path that will later be referred to as 'the axiomatic method' (cf. Shulman 1997, 429). This narrative conceptualizes *the human* and *the world* separate from said *human*. In this narrative, *the human* can access this different world through the epistemic endeavor described in the poem and later named 'the axiomatic method.'

In my examination of Parmenides' poem and the associated discourse, I am interested in the concept of *truth* and the *thinking* that promises to lead to this very *truth*. Such *thinking* promises to enable *the human* to access *the world* allegedly separated from him. My analysis identifies 10 aspects in Parmenides' poem, which I understand as the central elements of his image of thinking. All these aspects of Parmenides' image of thinking are interwoven with one another. As such, they are interdependent and mutually grounded – like a body in which one organ nourishes the other and keeps it alive.

Parmenides describes a lyrical self[6] that is led to a goddess (cf. ibid.). The goddess will reveal the truth and the path to truth to Parmenides. First, she welcomes him and describes the nature of what she intends to give him:

> It is no ill chance, but right and justice that has sent thee forth to travel on this way. Far, indeed, does it lie from the beaten track of men! Meet it is that thou shouldst learn all things, as well the unshaken heart of well-rounded truth, as the opinions of mortals in which is no true belief at all.
> *Parmenides' didactic poem, trans. by John Burnet (2005), 197*

Parmenides is greeted here by the goddess as she announces that she will show him the *truth* and teach him to recognize it. What interests me most in this passage is how she characterizes *truth and in particular,* which demarcations and promises go hand in hand with this great word. Reading with this question in mind already reveals some aspects that become increasingly relevant to Parmenides' image of thinking as the poem progresses:

The *truth* that the goddess wants to show Parmenides is universally valid; it is "the unshakeable heart of truly convincing truth." I consider this universalism to be the first characteristic of the Parmenidean image of thinking in which *truth* is *universally valid* knowledge.

6 For the sake of simplicity, I do not differentiate in my discussion between Parmenides and the lyrical self that he writes. I thus follow the conventional way of writing about Parmenides' didactic poem.

This universalism simultaneously establishes a demarcation of *truth* from *other knowledge,* and it is demarcated from "the opinions of mortals, in which there is no true reliability." *Truth* is here demarcated from *being human* and from the realm of *opinions* and *non-reliability. Resultingly,:* its universality feeds on its *non-humanness* – its *non-mortality* –its *sacrality*, is symbolized in the fact that *truth* is achieved through a goddess as an explicitly *non-human* figure.

This demarcation also reveals a fundamental binarity that continues to permeate the poem. This binarity consists in the idea that there exists the realm of *mortal, transient opinion* and that of *eternal truth* in which both spaces appear as worlds that only become what they are in opposition to each other.

This binarity is not simply an opposition, but specifically a hierarchized binarity. Parmenides' pursuit of *irrefutable truth* is explicitly named by the goddess as a *good* endeavor – as a "divine providence" and even as his "right." In this, the question of *truth is* already given an explicitly moral dimension by opening up the idea of a *morally good* sphere of *eternal truth*.

All these aspects are ultimately based on the organ that presumably represents the heart of the Parmenidean image of thinking: the idea that *truth* feeds on *necessity* –precisely that which cannot be otherwise is *true.*

The goddess continues to give Parmenides hints for the path to this *truth:*

> Come now, I will tell thee […] the only two ways of search that can be thought of. The first, namely, that *It is*, and that it is impossible for it not to be, is the way of belief, for truth is its companion. The other, namely, that *It is not*, and that it must needs not be,—that, I tell thee, is a path that none can learn of at all. For thou canst not know what is not—that is impossible—nor utter it; for it is the same thing that can be thought and that can be.
> *Parmenides' didactic poem, trans. by John Burnet (2005), 197–198*

This excerpt describes precisely one of the two central basic ideas of the axiomatic method as a *path to truth*. Namely, it shows that certain, *necessary principles count* as *foundations of truth* in that they are themselves *necessarily true*. The goddess in her subsequent sentences therefore states the following *axioms*: the *truth of being* and the *falsity of non-being*.

The idea of necessity, which has already been mentioned, is woven into this idea of the *axiom*. The *axioms* form the basis of the epistemic body because they are understood as *necessary.* This and this alone justifies their role. This aspect is illustrated, for example, through the goddess' emphasis that the two propositions presented, and their consequences are the only way of investigating *truth*.

This emphasis already hints at a link between *truth* and authority, which will become clearer as the poem progresses.

After her introduction to the character of *truth* and the method of the path to it, the goddess carries out the area opposite to that of *truth* – that of *error*:

> I hold thee back from this first way of inquiry, and from this other also, upon which mortals knowing naught wander two-faced; for helplessness guides the wandering thought in their breasts, so that they are borne along stupefied like men deaf and blind. Undiscerning crowds, in whose eyes it is, and is not, the same and not the same? and all things travel in opposite directions!
> *Parmenides' didactic poem, trans. by John Burnet (2005), 198*

Here the link between *truth* and *power* and thus the dimension of authority becomes explicit. *People* who are not in possession of the *truth* or aware of the path to it are affected by "powerlessness" precisely because of the absence of *truth* in their world. Indeed, they are *powerless because* they *are mistaken*.

And they *are mistaken* insofar as they do not observe the laws of binarity and contradiction: "peoples for whom being and non-being are the same and not the same again." This sentence reinforces the link between *truth* and *binarity* by all the more firmly establishing the idea of *binarity* as the path to *truth*.

This scene also deepens the moral dimension of *truth* and its differentiation from *mortals* (*humans*). *Humans* are described as those who are usually without *truth*, because this is actually, in a sense, above them – that is, above all – even above their mortality – in immortality. It is precisely in this characteristic that they are explicitly devalued, for example in the phrase "the double-headed." Furthermore, this de-valuation is portrayed through an ableist metaphor of 'blindness' as a stand-in term for 'stupidity' deepening the exclusionary language on modes of perception and thinking.

Finally, I would like to note the phrase "in a rational way." The "rational way" appears here as that which is conducive to finding the *truth*. I would like to note the conceptualization of *Rationality* as a path to *truth* as a further aspect of the Parmenidean image of thinking.

In conclusion, Parmenides' divine teacher focuses on the character of being as *truth*:

> One path only is left for us to speak of, namely, that *It is*. In it are very many tokens that what is is uncreated and indestructible ; for it is complete! immovable, and without end. Nor was it ever, nor will it be; for now *it is*, all at

> once, a continuous one. [...]
> Therefore must it either be altogether or be not at all. Nor will the force of truth suffer ought to arise besides itself from that which is not.
> *Parmenides' didactic poem, trans. by John Burnet (2005), 199*

This scene reveals the last aspect of the Parmenidean image of thinking that I would like to examine. This aspect is also the second decisive criterion of the axiomatic method, as it will be treated and applied later, following Parmenides (cf. Shulman 1997, 432): the idea of *seclusion*. The idea of seclusion appears here in two senses. The goddess describes *truth* as a system that is self-contained and is secluded in itself – separate other systems. This conceptualization of *truth* as a self-contained system is an essential organ of the Parmenidean image of thinking. Parmenides' teacher emphasizes the *complete immutability* of this system, from which its *necessary truth* is supposed to feed. Accordingly, this conceptualization turns separability into a logic through which to seek *truth* and 'epistemic certainty.' This system of truth also consists of aspects of the system insofar as individual truths all have an inner unity and yet are connected as a system.

In light of this analysis, I therefore summarize the Parmenidean image of thinking, which is often referred to as the *Mathematical image of thinking* (cf. Steuben 1981, 67 and Shulman 1997, 430), through reference to ten core claims[7]:

1) *Truth* appears in Parmenides' poem as 'universally valid knowledge.' This establishes the concept of an a-historicity as definitive for *truth*. Given that Parmenides' poem is received and receives uptake as a characterization of *Mathematics*, this aspect extends to Mathematics. As such, mathematical

7 The aspects that I list here result from on the one hand a close reading of Parmenides' didactic poem and on the other hand their recognition in the established discourse on his teachings. Therefore, each aspect is followed by an exemplary reference to secondary literature, which is in addition to the references to the poem itself and can be read as exemplary of the relevant research discourse. The respective points can be found as examples in the following secondary literature: Re 1): Nye 1990, 17/Shulman 1997, 427/, On 2): Nye 1990, 16/Steuben 1981, 104, Re 3): Nye 1990, 17/Shulman 1997, 433/ Steuben 1981, 108, Re 4): Shulman 1997, 432/Steuben 1981, 131, Re 5): Nye 1990, 11, 19/ Shulman 1997, 430, Re 6): Nye 1990, 11/Shulman 1997, 431, Re 7): Nye 1990, 17/Steuben 1981, 42, Re 8): Nye 1990, 12/Shulman 1997, 429, Re 9): Nye 1990, 12/Shulman 1997, 432, Re 10): Shulman 1997, 430/Steuben 1981, 111

thinking is constructed as the path to truth and to 'universally valid knowledge.'

2) This 'universal validity' is construed as secured through *epistemic necessity*. This means that a-historicity and universal validity is not only ascribed to *Mathematical knowledge*, but also to the intellectual technique described as *Mathematics* in the poem.
3) *Truth* is explicitly related to power insofar as it is the knowledge of special authority. This too is related to *Mathematics*, because *Mathematics* as an image of thinking, has been constructed with reference to this poem. *Mathematical knowledge* is thus knowledge with distinct epistemic authority – as we have already witnessed in the work of Pinker, Ellenberg, and Brooks.
4) *Truth* itself, as well as the search for it, has a moral dimension. Truth and its pursuit are honorable and *good*. This establishes a normative, moral dimension of *Mathematics* too; insofar as *Mathematics* is understood as a path to epistemic certainty it is also a morally superior path and a noble intellectual endeavor.
5) *Truth* (because of its eternity) lies outside the actual sphere of *human beings* and it is explicitly demarcated from *humans*. The method described in the poem – *the Mathematical image of thinking* – is the way for humans to access what is outside of them –*truth*.
6) *Rationality* is conceptualized as the *thinking* that leads to *truth* and as the intellectual method which brings *the human* to the *eternal truth*.
7) Parmenides constructs *truth* in relation to the concept of 'a system.' Truth is understood as a closed, eternally existing system of unchanging propositions. This implies that truth is conflated with a logic of separability, because 'truth as a system' is construed as epistemically believable precisely because it remains enclosed and separated from other systems.
8) There are *necessarily true principles* that must be assumed on the path to *truth, i.e.,* –what is later termed 'axioms.' Axioms are statements characterized as being *necessarily true*. *Axiomatic Mathematics* is therefore rooted in the very notion that there is *necessarily true knowledge,* from which further knowledge can be derived by means of *Mathematical necessity*. The notion of *(Mathematical) necessity* identifies the notion of the axiom and thus identifies *Mathematics*.
9) The individual sentences of the system that *truth* is – are understood as self-contained. The sentences making up the truth-system are separated from one another.

10) The path to *truth* emerges as a path of *binary logic*. *and contradictions* are to be recognized as 'a symptom of error.' The notion of contradiction as a signifier of error is constructed through a binary logic, as only in a two-value system (a system, where things are either *true* or *false*) can contradiction count as a signifier of falsity. So, thinking in binary logic is construed here as the only reliable path to truth and it is constructed as a necessary part of thinking.

Some of these aspects of *Mathematics* as an image of thinking – such as *Mathematical knowledge* as knowledge with special authority or the moral dimension of epistemic certainty – have already been observed above in the works of Pinker, Ellenberg, and Brooks. These aspects have thus sustained themselves for a long time and bear a transhistoricity as they move through Ancient Greek work and contemporary works – albeit not in the same way, but in an inherently related way.

As the second voice of Ancient Greek philosophy, I consider the work of Plato. Despite differences to the work of Parmenides, Plato takes up central features of his thought and expands and deepens significant moments of the Parmenidean *image of (Mathematical) thinking* (cf. Sworder 2013, 10).

In contrast to Parmenides, Plato himself explicitly addresses *Mathematics* in his philosophy and basic features of Plato's perspective show important correlations with the current prominent understanding of *Mathematics* (cf. Lassere 1963, 7).

Plato's work is much larger in scope than what is left of Parmenides' philosophy. Plato's understanding of *Mathematics* does not appear in a specific work as one comprehensive theory or position (cf. Sworder 2013, ix). My discussion of Plato will therefore draw on various dialogues and be oriented towards the works and quotations that are particularly frequently used in the discourse on Plato's *Mathematics*. I will focus on the part of the *Politeia* in which Plato first talks explicitly about *Mathematics* and then goes on to make use of the allegory of the cave, which Plato's teaching is associated with in a particularly dominant way (cf. Sworder 2013, 35/Politeia 510 c to 530 a).

Some crucial elements of the Parmenidean image of thinking are directly adopted by Plato and are therefore only mentioned here. This is because my examination centers the elements that are extensions or changes in comparison to the Parmenidean image of thinking[8]. Plato retains Parmenides' idea of a *necessary truth* (cf. Sworder 2013, 10 and 27/e.g. Politeia, 511 b) as well as its identification with something divine (cf. Sworder 2013, 114, 115). This is accompanied by the description of a fundamental insight from which everything else is derived (the *axiom*) and which is itself the result of an 'intuition,' rather than an effort of thought (cf. Sworder 2013, 27).

The summary of the Platonic image of thinking is particularly complex due to the large volume of work and the ideas on *Mathematics* that are repeatedly scattered throughout Plato's work. Therefore, I also consult secondary literature. The aspects discussed in the passages I have chosen are thus shown to be those that can be understood as a fundamental component of Plato's understanding of *Mathematics.*

First, I will establish the main features of the Platonic theory of ideas and its relationship to Plato's image of *Mathematical thinking*. Plato's *allegory of the cave* is used – both in Plato's work and in reference to it to illustrate Plato's theory of ideas. In the allegory of the cave, Plato describes people bound to a cave so that they can only ever see the shadows cast on the wall they face – the shadows of the things moving behind them. The shadows and the originals that cast the shadows are representative of the relationship between an idea and one of its cases (cf. Politeia Book 7, 514a to 515c, Sworder 2013, 9). The *idea* here is the *truth,* and the 'instances of the idea' participate in the idea to varying degrees (cf. Deleuze 1992, 90).

The *ideas* as a concept have a similar function to the notion of *being* in Parmenides' work and are closely related to this concept as they are *the true being* that receives its *truth* through its *eternity* (cf. Shulman 1997, 435).

This idea is brought in relation to 'mathematical entities' by Plato himself as well as others. Thus, the idea that mathematical objects lie in a separate, eternal sphere and therefore have a specific ontological status bearing the name *mathematical Platonism*. Contemporary mathematicians, such as Marcus du Sautoy, refer to this idea with conviction:

> What about the question of whether mathematical objects really exist? I certainly am a Platonist at heart. There are some things out there that are inde-

8 Plato further develops Parmenides' method of division in the dialogue *Timeon*. Due to the explicit focus on *Mathematics* here I do not include *Timeon* in my re-readings of Plato. However, it can be helpful to note that Plato draws from Parmenides more explicitly in *Timeon*, especially further establishing Parmenides' method of division.

> pendent of our existence or act of imagining them.
> *Du Sautoy 2011, 23*

This notion of a specifically mathematical ontological status borrows directly from Plato's theory of ideas and is linked above to a specific claim and concept of *truth*, which consists in the postulate of *independence*. Ideas are *true* because they are not in the sphere of *human life* and the measure of perfection of the Idea can never be fully attained by its cases (cf. Deleuze 1992, 90 and Sworder 2013, 9). In precisely this sense, ideas are ideals, which consist precisely in the fact that they cannot be attained (cf. ibid.). Thinking in terms of mathematical Platonism, this applies precisely to mathematical objects or *Mathematics* as a sphere – a sphere in which it is the ideal of thought to which all other thought aspires due to the premise that this aspiration can never be completely fulfilled (cf. ibid.).

In the context of his theory of ideas, Plato explicates another element that we today naturally associate with *Mathematics*, and possibly with *thinking* in general: the element of *abstraction* and specifically its connection with the idea of *universal validity*.

> In the 'abstract forms' of contemporary mathematics we recognize the axioms of Euclid's predecessors, and in the 'aspects of experiential reality' Plato's mathematical universe.
> *The Birth of Mathematics in the Age of Plato by Francois Lasserre (1964), 14*

For Plato, *Mathematics* is part of the description of the world of ideas that exists in the realm of the *abstract* and the *non-worldly.* – it is that which passes through the world without existing in it as an explicit, recognizable object (cf. Shulman 1997, 434). And precisely this property of abstractness appears not only as an ontological property, but also an epistemic one. Thus, the character of abstractness is what *universal validity* is supposed to result from, "[…] seeking to see those other things themselves that one cannot see except by means of thought." (Politeia 510c 5).

In this section, Plato distinguishes between four different spheres of human understanding and understands *Mathematics* as part of the sphere (*noesis*) that "can only be seen through thinking" (cf. ibid. and Sworder 2013, 27).

In the context of the postulate of completeness and abstraction, Plato deepens the idea of *truth as a system,* which was already recognizable in Parmenides. Plato's world of ideas consists of different ideas that are related to each other and that together form a self-contained system that is complete for all eternity. Plato writes the following in the dialog *Menon:*

> As the whole of nature is akin, and the soul has learned everything, nothing prevents a man, after recalling one thing only—a process men call learning—discovering everything else for himself, if he is brave and does not tire of the search, for searching and learning are, as a whole, recollection.
> *Plato, Meno, trans. by G.M.A. Grube (1997), 81c 9 – d 4*

Various central aspects of Plato's doctrine become clear through his presentation of the world of ideas as one that is given to *all human beings. In doing so,* he also links the experience of mathematical entities and order, insofar as they are part of the world of ideas, with human experience (cf. Sworder 2013, 10 and 27). The ideas he inputs or ideas are what I have previously described with the term axiom, i.e., they are what *everything else*, that is all *possible knowledge* and the entire functioning of the system, can be derived from (cf. Sworder 2013, 27):

> He makes the experience of being the perception of a single aspect of a complex system of eternal unchanging ideas.
> *Mathematical Plato by Roger Sworder (2013), 27*

Here Plato deepens in several respects the axiomatic image of thinking that was already recognizable in Parmenides at the beginning. According to this concept, the search for *truth* is based on certain *unchangeable* and *unquestionable* propositions from which a system is formed that determine the limits of validity.

The relationships within this system are those of *logical necessity* and their investigation and their determination is the task of *Mathematics* (cf. Sworder 2013, 27). Philosophy, on the other hand, has the task of determining and evaluating the unquestionable propositions – the axioms (cf. Sworder 2013, 38):

> [B]y the other subsection of the intelligible, I mean that which reason itself grasps by the power of dialectic. It does not consider these hypotheses as first principles but truly as hypotheses—but as stepping stones to take off from, enabling it to reach the unhypothetical first principle of everything. Having grasped this principle, it reverses itself and, keeping hold of what follows from it, comes down to a conclusion without making use of anything visible at all, but only of forms themselves, moving on from forms to forms,

and ending in forms.
Plato, Republic, trans. by G.M.A. Grube and C.D.C Reeve (1997), 511 b

Philosophy and *Mathematics* therefore both move within the axiomatic system and they both move as the world of *truth*, yet they and up different positions in their respective explorations.

This link between philosophy and *Mathematics* is a central aspect of Plato's teaching. In the academy he opens, there is a prominent focus on the teaching of *Mathematics* and *Mathematics* is also an explicit and significant part of the program in his concept of 'the good state,' outlined in the *Politeia* (cf. Lasserre 1967, 27). So, as I shave shown in Pinker above, there is a political dimension –a political hope to *Mathematics*. *Mathematics* is a central component of the development of *the whole of truth* and is described by Plato as 'the highest form of fulfillment,' and as a thinking with special merit, special significance (cf. Sworder 2013, 10).

In many respects then, Plato already implies the interwovenness of *Mathematics* and *being human* – so the *Mathematics-Human* continuum observed in Pinker and Brooks, also appears in Plato in its own way. Plato understands *Mathematics* as 'the greatest fulfillment of human passion' and the central movement in the search for *truth*, the fulfillment of which is understood as a deep, and distinctly *human* need (cf. ibid.). These are instances of the *Mathematics-Human* continuum in Plato as well as the concept that constructs *Mathematics* and *being-human* as being a part of one another. as understanding *Mathematics* as a fulfillment of a distinctly human desire, is to identify *Mathematics* as an inherent part of *the human*.

I summarize the Platonic *image of (Mathematical) thinking* in five core claims. The Parmenidean image of thinking was not yet linked to *Mathematics* by Parmenides himself, whereas attribution to *Mathematics* explicitly applies to all the following aspects of Plato's philosophy[9]:

9 The respective points can be found as examples in the following secondary literature:
Re 1) Shulman 1996, 433/Sworder 2013, 27
Re 2) Nye 1990, 24/Sworder 2013, 28
Re 3) Nye 1990, 37/ Shulman 1996, 434
Re 4) Lassere 1964, 7/ Shulman 1996, 433/Sworder 2013, 27, 32
Re 5) Nye 1990, 30/Sworder 2013, 21–22

1) Plato deepens the idea of the *axiom* as an infallible, fundamental premise and relates it directly to *Mathematics*. The notion of the *axiom* arises from the idea of *epistemic necessity*. It is the very notion of *epistemic necessity* that constructs *Mathematics* and is in turn constructed by *Mathematics* as an image of thinking.
2) The deepening of *truth* is a perfect, self-contained and eternal system. Individual elements are linked to one another through 'logical relationships,' and relationships of *necessity*. This further establishes the centrality of *necessity* to *Mathematics*. Also, this aspect establishes the logic of separability as a part of *Mathematics* as an image of thinking because the notion of the self-contained system relies on of a logic of separability, *self-containment* is to be *separated* from another thing.
3) Plato explicitly constructs *Mathematics* and the idea of *the human* in mutual relation: Showcasing one of the central instances of the *Mathematics-Human* continuum in Plato, *Mathematics is constructed* as the knowledge, the thinking, which 'all human can recall' and that brings 'the highest human fulfillment.
4) Plato links *abstraction* and *universality* and classifies *Mathematics* as *abstract* and *universally* valid. This explicit conflation of *the universal* and *the abstract* is in comparison to Parmenides' philosophy, but is familiar to that done by Pinker, Ellenberg, and Brooks.
5) Plato's teaching deepens the idea of *eternal concepts* through his theory of ideas. This notion of eternal concepts is explicitly identified with *Mathematics* because *Mathematical entities* are understood as eternal concepts in Plato's work.

Each of these elements, which still exist today, are part of the axiomatic system, that emerges in Plato's time and in explicit reference to Plato (cf. e.g. Shulman 1996, 433). Thus, the notion and practice of the axiomatic system lives a transhistorical life– it moves through different historical periods, not by being constant, but by adapting its core characteristics to its specific existence.

I examine the work of Aristotle as my last stop in my travels through Ancient Greek philosophy. Unlike Plato or Parmenides, Aristotle's work contains a section that is decidedly devoted to the design of a mathematical ontology that take up from Plato and at the same time deviates fundamentally from his 'doctrine of the heaven of ideas' (cf. Lear 1982, 161). Aristotle's work on *Mathematics* is situated as an ontological work, a study of being. This is crucial, because both Plato and Parmenides have moved on a primarily epistemic, or thinking-oriented level, while Aristotle studies *Mathematical entities* as *beings*. As I have shown in my examination of Pinker, Ellenberg and Brooks there is a *thinking-being* continuum present in *Mathematics* and in the *Mathematics-Rationality-Human* continuum. So, the study of Aristotle will partly be devoted to studying the relation of *thinking* and *being* in Aristotle's account of *Mathematics*.

To begin, I focus on Aristotle's philosophy of the ontological status of *Mathematical objects* in order to explore the question of how Aristotle understands *Mathematical truth*. My second focus is then his method of *syllogism, which* is a method of demonstrating *immutable truth* (cf. Nye 1990, 52). Aristotle applies the latter not only to *Mathematics* but understands it as a philosophical logic whose application in *Mathematical* contexts is of particular merit within his philosophy. I first provide an explanation of the main features of Aristotle's mathematical ontology and syllogism by analyzing selected parts of the writings *the Categories* and *Metaphysics*[10].

Aristotle's philosophy of Mathematics begins with an ontological question that he poses following Plato. Plato asserted the claim of *Mathematics,* as the sphere of the *immutably true knowledge,* by locating it in a sphere of its own – that of 'the heaven of ideas' (cf. Lear 1982, 161). Aristotle understands this explanation as unsatisfactory in the sense that it is unable to explain why statements about the physical world can be made based on *Mathematical objects* (cf. ibid.). The dilemma is the following: if mathematical objects exist in a world that is not that of material objects then how can we believe that *Mathematics* can produce *truth* in relation to the world of material objects?

Thus Aristotle, like Plato, wants to assert that *Mathematics* can produce knowledge about the *immovable* and the *unchanging,* and wants to explicitly dispense with the idea of a *mathematical heaven of ideas in* his explanation of how this can be possible (cf. ibid.). Accordingly, from the very starting point of Aristotle's philosophy of *Mathematics* there is a conflation of *thinking* and *being*. Aristotle studies *Mathematics* from an ontological standpoint to explain the epistemic status of *Mathematical knowledge* and he studies *being* to speak on *knowing*.

The basic ideas that Aristotle outlines in this project are still largely relevant today for both *mathematical-philosophical* questions and conventional ideas

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10 In the selection of text passages, I stick to those that are quoted particularly frequently in the context of Aristotle's philosophy of mathematics and his concept of syllogism, which are considered the relevant excerpts on these questions. My central references in the selection are Nye 1990, Lear 1982, Gillies 2015.

about why *Mathematics* is capable of producing reliable knowledge (cf. Gillies 2015, 146). So let us first investigate the main features and basic theses of Aristotle's project:

> It has, then, been sufficiently pointed out that the objects of mathematics are not substances in a higher degree than bodies are, and that they are not prior to sensibles in being, but only in definition, and that they cannot exist somewhere apart.
> *Aristotele, Metaphysics XIII Book (M) trans. by Jonathan Barnes (1984), 1077b*

Here Aristotle begins by stating two of his central starting points. Firstly, he assumes that mathematical objects exist. Secondly, he assumes that they do not do so in their own separate sphere. The existence of mathematical objects is a central conviction for him, which he subsequently emphasizes further:

> [...] it is true also to say without qualification that the objects of mathematics exist, and with the character ascribed to them by mathematicians.
> *Aristotele, Metaphysics XIII Book (M) trans. by Jonathan Barnes (1984), 1077b|1078a*

From here Aristotle poses the question as to how and in what way mathematical objects exist:

> [...] so too is it with geometry; if its subjects happen to be sensible, though it does not treat them qua sensible, the mathematical sciences will not for that reason be sciences of sensibles-nor, on the other hand, of other things separate from sensibles.
> *Aristotele, Metaphysics XIII Book (M) trans. by Jonathan Barnes (1984), 1077b|1078a*

In this section, Aristotle develops one of the central theses of his theory on the status of being and truth and thus claims of *Mathematics*, which is referred to as *the thesis of embodiment* (cf. Gillies 2015, 152): Mathematical properties and objects are said to have real existence without being able to exist independently of material objects. However, Aristotle does not reduce mathematical objects to their so-called accidentals, i.e. the material objects in which they occur, but distinguishes mathematical objects from their material accidentals.

These ontological questions are of central importance for Aristotle's view of *Mathematics* and its claim to truth as it is nothing less than an ontological investigation of the question of whether and, if so, why *reliable knowledge* can be expected from *Mathematics* and of what character, and above all, what status of reliability and *Truth*, this knowledge can attain (cf. Lear 1982, 162). Aristotle explains this connection in the following section:

> Each question will be best investigated in this way-by setting up by an act of separation what is not separate, as the arithmetician and the geometer do. For a man qua man is one indivisible thing; and the arithmetician supposed one indivisible thing and then considered whether any attribute belongs to a man qua indivisible. But the geometer treats him neither qua man nor qua indivisible, but as a solid. For evidently the properties which would have belonged to him even if perchance he had not been indivisible, can belong to him even apart from these attributes. Thus, then, geometers speak correctly; they talk about existing things, and their subjects do exist; for being has two forms-it exists not only in complete reality but also materially.
> *Aristotele, Metaphysics XIII Book (M) trans. by Jonathan Barnes (1984), 1078a–1078a|1078b*

Here Aristotle introduces a distinction that is essential to his view of *Mathematics*: a material entity has properties that *necessarily exist* – for example, those that are common to all *human beings* insofar as they are *human beings* – and those that also exist by chance and can therefore vary (cf. Nye 1990, 57). Aristotle understands *Mathematics* as the determination and investigation of *essential* and thus *universal properties*, which are *immovable* and *unchangeable. Mathematics* is the *deduction of truth* insofar as it carries out the investigation of these very properties.

Similar to Parmenides and Plato, Aristotle's image of *Mathematics* also carries the figure of *necessity* and *completeness* deep in its core (cf. ibid.). As Aristotle writes at the beginning of the section, (mathematical) investigation is not one kind of knowledge among many, but the *best possible kind*.

So let us note the following about Aristotle's ontological conception of *Mathematics* and the *truth* it produces. Firstly, mathematical objects exist in the form of unchanging, essential aspects of the material, physical world and *Mathematics* is thus the study of what cannot be otherwise (cf. Lear 1982, 163, 183).

In order to focus on Aristotle's methodological conception in the context of the *search for truth* from this ontological concept, I will at this point take a look at Aristotle's concept of the *syllogism*. He outlines the same concept in the *First* and *Second Analytics,* not referring exclusively to *Mathematical Knowledge,* but outlining the very rules that are also those of modern mathematical reasoning (cf.

Lear 1982, 162). So let us look at Aristotle's syllogism in order to understand how *people*, according to his image of *(Mathematical) thinking*, gain access to *necessary knowledge*. The following section provides an initial insight into Aristotle's idea of knowledge:

> When the extreme terms are convertible, the middle term must be convertible with each of them. For if A is true of C because B is A and C is B, then if All C is A is convertible, All C is B, All A is C, and therefore all A is B, and All A is C, All B is A, and therefore all B is C.
> *Aristotele, Prior Analytics trans. by Jonathan Barnes (1984), 67b 27*

Here Aristotle formulates the idea of *knowledge as a system of axioms* as indicated by Parmenides and Plato. He contends that there is a premise (an axiom), in this case "A is present at that at which B is present", and from this follows further necessary knowledge, "that C is present." Let us pursue this idea further by returning to Aristotle:

> and when one's inference is derived only from the possible one cannot be said to have knowledge in the true sense of the word. When the conclusion is necessary there is nothing to prevent the middle term, by means of which the conclusion was proved from being necessary, for it is possible to infer the necessary from the not necessary, just as one may infer the true from the untrue.
> *Aristotele, Posterior Analytics trans. by Jonathan Barnes (1984), 1.6*

Here Aristotle formulates the idea of an *axiom in which there* is a knowledge that is *unprovable* and requires no provability because it *necessarily* exists. *Provable knowledge* is derived from this *unprovable, necessary knowledge*. The strategic conception of *provability* is based on a specific role of the idea of *contradiction* in which *contradiction* is regarded as a symptom of *falsity* and is what makes a *proof* fail (cf. Nye 1990, 50). Aristotle further formulates the idea of *unprovable knowledge that does not require proof* along the lines of the category of *man:*

> if then we have no higher perception of the demonstrable than knowledge, the result must be that we cannot know anything absolutely by means of demonstration, but only conditionally.
> *Aristotele, Posterior Analytics trans. by Jonathan Barnes (1984), 1.22*

Aristotle here establishes the idea of *universal, human perception,* which encounters 'the necessary bodies of knowledge' that can be taken for granted. The general *truth* lies in perception and would be recognized with *reason,* and in such a way that from then onwards it constitutes *irrevocable knowledge* from which *evidence* of other knowledge can be provided. This idea links *reason* directly with *knowledge* and *knowledge* in turn with the idea of *provability*, which takes place strictly within the concept of the axiomatic system. For Aristotle, a proof is the demonstration of knowledge that exists independently of itself.

Aristotle departs from those elements in the philosophy of Parmenides and Plato that appear mystical or metaphysical, while retaining and intensifying their claim to knowledge of *the immovable*, the *unchangeable,* formulating it into a method that is explicitly identified with the category of *man* and the distribution of political power. To summarize, Aristotle's image of *Mathematical thinking*, is captured in the following core claims[11]:

1) *Mathematical knowledge* is knowledge about the material world that *necessarily* exists. In this sense, *Mathematical knowledge* is knowledge with an outstanding authority that is precisely established through the idea of necessity. So, as seen in Plato as well as Ellenberg, the special epistemic status of *Mathematics* is explained through the identification of *Mathematics* with *necessity*.
2) Mathematical knowledge has two forms: the *unprovable, necessary knowledge* (axioms) and the *provable knowledge,* which becomes provable through the *unprovable knowledge* or the axioms. So, there are two ways knowledge can have this special epistemic authority attributed to *Mathematics*: either it can qualify as an *axiom,* or it can qualify as *provable*.
3) *Unprovable knowledge* stems from a specific, universal part of perception that is discovered with *reason*. As in the work of Aristotele, Pinker too conflates universalism and reason with one another, which highlights another transhistorical aspect of the *Mathematics-Rationality-Human* continuum.

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11 The respective points can be found as examples in the following secondary literature:
Re 1) Gillies 2019, 23/ Lear 1982, 164/Nye 1990, 461
Re 2) Lear 1982, 172/ Nye 1990, 58
Re 3) Lear 1982, 169, 183/Nye 1990, 57
Re 4) Lear 1982, 164/Nye 1990 1990, 58/Shulman 1996, 435
Re 5) Lear 1982, 172/Nye 1990, 172/
Re 6) Nye 1990, 58/Shulman 1996, 435
Re 7) Nye 1990, 43/Shulman 1996, 435

4) A *proof* is the demonstration of a *truth* that exists independently of that proof. A proof has authority insofar as it is a demonstration and not the originator of the knowledge in question. This is crucial because the proven knowledge has epistemic authority because the proof is understood not to construct the proven knowledge, but rather, to merely point to it in a sort of gesture extending beyond the proof as a method. Furthermore, this aspect showcases how ontological questions intersect with questions of epistemic authority. as the ontological status of the knowledge in question is decisive for the level of epistemic authority attributed to that knowledge.
5) The *contradiction* is a symptom of the *invalidity* of a proof or the *falsity* of a statement. Similar to Pinker or Parmenides, this postulate introduces a logic of binary into *Mathematics* and *Rationality* as images and methods of thinking.
6) Aristotle deepens the idea of the *closed nature* of questions and answers and thus deepens the logic of separability built into *Mathematics*. As such, meaningful questions and answers are precisely those that take place in the axiomatic system and in the logic of a separability in seclusion from one another.
7) Aristotle's concept of *proof* and *reason* are explicitly characterized in a political dimension. A human who moves validly within the axiomatic system, i.e., in accordance with its logics of thinking-being, is a person to whom epistemic and political authority is to accrue. This political dimension of the notions of proof and reason were also observable in Pinker's work and has thus lived transhistorical lives as well.

Parmenides, Plato, and Aristotle all differ from one another in their teachings and yet they can all justifiably be understood as founding figures and works of the axiomatic image of thinking, of Mathematics as an image of thinking-being. Their common core and the project they all pursue – albeit all differently – is that of a knowledge that is *immutable* and to which *infallible access* can be granted. Plato and Aristotle share an explicit construction of *Mathematics* in precisely this context in their understanding of *Mathematics* as a path to this *immutable, infallible knowledge, which* is particularly infallible because of the characteristic of *necessity* built into *Mathematical knowledge* and *Mathematical thinking*.

Aristotle establishes the role of *Mathematics* in the strongest and most concentrated way as the knowledge of 'a special claim to truth.' However, all three works contribute to an identification of *Mathematics* with *necessity*, which is based specifically on the notion of axiomatic reasoning.

In both Aristotle and Plato, *Mathematics* is identified as 'a universally human perception' making the *Mathematics-Human* continuum a shared aspect of their understandings of *Mathematics*.

The traits of the *image of (Mathematical) thinking* shared between all three works can be summarized as follows:

1) They are guided by the notion of a search for *unchanging, universal truth* linked to the idea of *necessity*: the notion that there is *knowledge* that cannot be otherwise, and that this knowledge is *Mathematical knowledge* precisely because of the conflation of epistemic necessity and *Mathematics*.
2) *Mathematics* is construed as a sphere in which this rigorous claim to a *necessary truth* is considered to be fulfilled. Because of this attribution of *epistemic certainty*, a particular epistemic authority is granted to *Mathematics*.
3) The idea of *necessary truth* is conceptually interwoven with the axiomatic image of thinking. It is believed that certain knowledge is *necessarily true* without proof, and that further knowledge can be inferred with intellectual methods that grant it *epistemic necessity*. In the axiomatic image of thinking all knowledge is therefore *necessarily true knowledge*.
4) *Truth* is understood as a system that is connected within itself by relationships of *epistemic necessity* and is unalterably closed to the outside world and this brings a logic of *separability* into *Mathematics* as an image of thinking.
5) *Contradiction* is construed as an indicator of epistemic *falsity* thus establishing a logic of *binary* within *Mathematics.*
6) *Mathematical knowledge* is understood as the knowledge to which all humans have access, so that *Mathematics* becomes part of the definition of *being human* and the other way around, which establishes the *Mathematics-Human* continuum.
7) The thinking that produces *true knowledge is* called *reason* and *Mathematics* is understood as an especially strong and triumphant example of *reason*.

In this sense, Ancient Greek philosophy can be understood as the birth of *Mathematics* as an image of thinking. This points to the Western character of *Mathematics* and to the political dimension of the question of *Truth,* which is evident in the explicit interwovenness of *Mathematics* with a special claim to epistemic authority. To further trace the historical makings of *Mathematics* and the *Mathematics-Rationality-Human* continuum, I will now turn to the philosophy of Enlightenment and its relation to *Mathematics*. This is not to denounce the existence of non-*Mathematics* forms of mathematical knowledge making – such as Indian, Babylonian, Chinese or Amerindian mathematical practice. But, because all mathematical systems in non-accordance with *Mathematics and* with the *Mathematics-Rationality-Human* Continuum, have been made to inhabit the place of 'the other' in relation to the universalized Westernized mathematical practices.

1.4 The Expansion of *Mathematics*
Rationality and *the Human* in the Philosophy of the Enlightenment

In my examination of the Enlightenment, I draw from the philosophies of René Descartes and Immanuel Kant. Both philosophers represent different philosophical traditions and yet are repeatedly referred to as central philosophers in the conception of *human reason* and also shape the conventional understanding of the *Enlightenment* period to a particular extent (cf. e.g. da Silva 2017, 6). I am interested in how they each develop their epistemic claims and practices and what role their understanding of *Mathematics* plays in their respective understanding of *thinking* and *reason*.

My consideration begins with Descartes, who is widely associated with his 'method of universal doubt' (cf. e.g. Tarek 2020, 25). This method requires doubting everything one believes in order to retain only that which can soundly withstand any doubt in order to arrive at 'certain knowledge' (cf. Brown 1980, 23). This methodological proposal reflects the fundamental claim of Descartes' work, which Gilles Deleuze describes as 'the claim of a complete beginning' (cf. Deleuze 1992, 169) in which *nothing* in knowledge or belief is simply to be retained and *everything* is to be doubted and tested (cf. Tarek 2020, 25).

Descartes was active as a philosopher as well as a *Mathematician*, whereby his philosophical claim sought realization through his *Mathematics* (cf. Felgner 2020, 115). I am particularly interested in the connection between his philosophical and *Mathematical* work and the conception of *Mathematics* that he develops in the process. Descartes' *Geometry* as his main *Mathematical* work, will only play a marginal role here, as Descartes establishes his core epistemic claims, his methods, as well as his understanding of *Mathematics,* primarily in the *Meditations* and the *Rules*[12] (cf. Tarek 2020, 2).

In the beginning of the *Meditations,* Descartes describes the methodological idea of a doubt that is related to all believed knowledge, in which a kind of 'epistemic new beginning' is created. Everything that is considered knowledge within the universal doubt is thus supposed to be 'knowledge beyond doubt.' Universal doubt should identify what cannot be doubted and what in turn can be doubted and must therefore be proven. This is a similar distinction as that of axioms and knowledge, which are necessarily derivable from the axioms as we have witnessed in the works of Aristotle and Plato; thereby, establishing another transhistorical aspect to *Mathematics*.

The epistemic aim that Descartes wants to fulfill is described with the concept of *truth*, which is associated with the idea of 'non-doubtable knowledge' and with the concept of *provability. Proving* is the epistemic practice that establishes the status of the *non-doubtable.*

In the *Rules* Descartes expresses that the aim of his philosophy is to develop a method by which all humans can recognize *truth* as such and know everything about it (Rules, 71). A conception of *truth* as a system is implied here insofar as Descartes clearly assumes the closed nature of *true knowledge*: by understanding *truth* as a closed system through which *everything* can be known – again evoking separability and seclusion.

To develop this method, Descartes establishes two epistemic practices in the *Rules*: *intuition and deduction* (Rules, 17).

Descartes selects the epistemic methods that he declares to be *valid* according to his epistemic claim regarding the search for *non-doubtable truth*. He assumes that specific epistemic practices methodically exclude deception, i.e. that certain practices of producing knowledge necessarily fulfill the claim to *absolute truth* and others do not. Descartes identifies two epistemic practices as those that realize his epistemic aim for 'knowledge beyond doubt': *intuition* and *deduction*.

12 There is debate as to whether the method in the *Meditations* is the same as that in the *Rules* (cf. e.g. Tarek 2020, 27). I therefore refer to both works equally and verify all aspects that are relevant to my work on the basis of both works. All aspects that I define as "Descartes' image of thinking" can be found explicitly in both works and are also recognized in research on Descartes as central aspects of his work.

He goes on to write about *intuition* as a method:

> By intuition I understand, not [...] the conception which an unclouded and attentive mind gives us so readily and distinctly that we are wholly freed from doubt about that which we understand. Or, what comes to the same thing, intuition is the undoubting conception of an unclouded and attentive mind, and springs from the light of reason alone; it is more certain than deduction itself, in that it; is simpler, though deduction, as we have noted above, cannot by us be erroneously conducted.
> *Descartes, Rules trans. by Elizabeth S. Haldane and G. R. T. Ross (2017), 22*

Here Descartes describes *intuition* as even more reliable than *deduction* in that it provides more direct access to the *truth*. *Intuition* describes the perception of the indubitable character of a certain knowledge or a form of *indubitable* insight into the *indubitable* character of a specific knowledge.

The idea of *intuition* signifies a form of universalization of perception insofar as it is based on *innate ideas that* are inherent to *all people* qua *being human*. A specific way of thinking is thereby already implicitly linked to *human perception* in general. Descartes uses two geometric and thus mathematical examples of such knowledge, which can be recognized in its *indubitability* through *intuition*. Geometric knowledge appears here as an example of *the most certain knowledge.*

Descartes characterizes the method of deduction as follows:

> [...] many things are known with certainty, though not by themselves evident, but only deduced from true and known principles by the continuous and uninterrupted action of a mind that has a clear vision of each step in the process. It is in a similar way that we know that the last link in a long chain is connected with the first [...].
> *Descartes, Rules trans. by Elizabeth S. Haldane and G. R. T. Ross (2017), 24–25*

Here Descartes elaborates on the relationship between *intuition* and *deduction*. Deduction is *necessary* because not all *certain knowledge* is intuitively recognizable, but *intuition* forms the principles from which and with which further knowledge can be deduced. This gives *knowledge* from intuition the character of an axiom: it represents the knowledge that is already certain and can be used to produce further knowledge.

This also makes clear what has already been hinted at, primarily, that Descartes understands *truth* as a system, as is particularly clear in his image of the chain. Accordingly, *truth as a system has* individual elements that are all connected to one another in a definable, recognizable way.

The concept of *intuition* comes from Descartes' *Rules* and is not used in the *Meditations.* In the *Meditations,* however, Descartes asserts a similar idea using the terms *clearly* and *distinctly* by establishing that everything 'seen clearly and distinctly' is to be recognized as *true* (cf. Meditations, 13–14, Felgner 2020, 115).

He makes this methodological proposal based on the consideration that those ideas that originate from our own imagination and only from our imagination must be *imperfect* and *incomplete. This is because* human beings, as *imperfect, fallible beings* cannot develop *perfect ideas. However, if* we nevertheless find perfect ideas in human beings, Descartes argues, they must be *innate*, God-given, *true* ideas (cf. Brown 1980, 27). These 'innate truths' are what is seen *clearly and distinctly*. Thus, a *true idea* to Descartes is true insofar as this idea has a specific form that is *unchangeable* and *eternal*. The identification of *immutability* and *eternity* with *truth* is clear here as what is *true* is precisely that which is not subject to change. *Truthfulness* does not depend on the physical occurrence of the idea or the thing that represents the idea.

Descartes establishes a notion of *provability* in this context. To *prove* that a triangle has certain properties is to see these properties *clearly* as those of the triangle, even though one knew nothing of them before the proof. According to Descartes, this process proves that the idea in question (here that of the triangle) contains properties that we have not added ourselves. This makes it a *true, innate idea*.

Descartes illustrates his rigorous epistemic claim of *eternal, complete truth* here along the lines of *Mathematics* and thus identifies certain *Mathematical Knowledge* as such *eternal, unchanging knowledge, which*, according to his vision, cognitive processes should strive for. *Mathematical knowledge thus* emerges as an exemplary fulfillment of Descartes' epistemic claim of 'unquestionable knowledge.'

In *Geometry,* Descartes lives out this philosophical claim in the sphere of *Mathematics:* His *geometry* strives to identify 'true geometrical ideas' and to create a geometry that operates exclusively with precisely these ideas (cf. Tarek 2020, 22). Descartes defines the *truthfulness* of geometric ideas in reference to another mathematical discipline by arguing that *truly geometric ideas* are those that can be expressed algebraically (cf. Felgner 2020, 118). For Descartes, *truthfulness* can thus be secured within *Mathematics* and through *Mathematics* itself. This again results in a special link between *Mathematics* and *Truth*.

The fact that Descartes repeatedly uses mathematical examples in the *Rules* and the *Meditations* and decides to implement his epistemic claim, developed in philosophy, in the sphere of *Mathematics* suggests that Descartes understands at least certain *Mathematical Knowledge* as a particularly successful example of 'secure, eternal knowledge.' *It* is precisely this stance that he puts into practice both in the *Meditations* and in the *Rules:*

> For whether I am awake or asleep, two and three together always form five, and the square can never have more than four sides, and it does not seem possible that truths so clear and apparent can be suspected of any falsity [or uncertainty].
> *Descartes, Rules trans. by Elizabeth S. Haldane and G. R. T. Ross (2017), 394*

Descartes distinguishes here between empirical knowledge as doubtful and apodictic knowledge (independent of experience), which is itself *transparent truth*. He cites two mathematical disciplines as examples of *certain, transparent truths* to which no imputation of *falsity* can be *meaningfully* attributed.

Mathematical knowledge appears here as an example of the category of the *most certain knowledge* and is specifically characterized by the fact that it cannot be meaningfully questioned, and the *truth* of the named *mathematical knowledge is* described as *plausible* and *necessary.*

In addition, the certain knowledge as which mathematical knowledge appears here is distinguished from the sphere of experience and "composite things." *Mathematical knowledge,* which Descartes describes here, acquires precisely this character of *undoubted truth* through its *eternity* and *immutability*, which leads to independence from physical occurrence.

Also noteworthy is Descartes' classification of "composite things" as objects about which knowledge is more uncertain. This remark follows his image of the chain and shows that Descartes sees his epistemic claim of establishing *absolute truths* fulfilled in a form of division. Thus, he here characterizes the consideration of the "individual parts" as the central and greatest prospect of *truth*. In this way, he understands a consideration of the *individual* to be more credible than that of the *coherent*.

Nevertheless, Descartes also applies the method of universal doubt, which is used in the Meditations, to *Mathematics*:

> And, on the other hand, always when I direct my attention to things which I believe myself to perceive very clearly, I am so persuaded of their truth that

> I let myself break out into words such as these: Let who will deceive me, He can never cause me to be nothing while I think that I am, or some day cause it to be true to say that I have never been, it being true now to say that I am, or that two and three make more or less than five, or any such thing in which I see a manifest contradiction.
> *Descartes, Rules trans. by Elizabeth S. Haldane and G. R. T. Ross (2017), 426*

Here Descartes once again secures specific *Mathematical knowledge* against doubt: even divine omnipotence cannot make this knowledge false.

What is new here is the mention of *contradiction* as a criterion as the reason Descartes gives for the irrefutability of the *truth of* this knowledge as otherwise a *contradiction* would arise. *Contradiction* naturally appears here as an indicator of *falsity.*

In the further course of the *Meditations,* Descartes secures *mathematical knowledge* even more globally and explicitly against his own *methodological doubt:*

> Even if not everything I have meditated on in the past few days were true, the existence of God in me would have to have at least the same degree of certainty as the truths of mathematics.
> *Meditations*, 71

Descartes here summarizes the *knowledge* of God and the *knowledge* of *Mathematical truth* as the *knowledge* to which *complete certainty* belongs. Descartes thus locates *Mathematical knowledge* outside the realm of *meaningful doubt* and inside the realm of *knowledge beyond doubt*.

Descartes creates an image of thinking that has the aim to establish *incontrovertible truth* at its core. Throughout this endeavor he repeatedly identifies *Mathematical knowledge* as an example of this kind of *incontrovertible truth* in various works.

Based on my own reading and the prominent research on Descartes, I summarize his *image of (Mathematical) thinking* through the following core claims[13]:

13 The respective points can be found as examples in the following secondary literature:
Re 1) Rozemond 2008, 41/Tarek 2020, 5
Re 2) Rozemond 2008, 41/Tarek 2020, 5/Felgner 2020, 119
Re 3) Bos 2001, 6/ Brown 1980, 24/Felgner 2020, 118
Re 4) Tarek 2020, 4/Felgner 2020, 119
Re 5) Brown 1980, 26/Felgner 2020, 113

1) Descartes strives for a *truth* that is characterized by *immutability* and *universality*. *This truth* is thus independent of both *human beings* and their physical occurrence. The notions of *immutability* and *universality* as characteristics of truth mark another similarity to the works of Parmenides, Plato, and Aristotle as well as to Pinker and Brooks – showcasing a transhistorical life to the mutual conceptual making of *truth*, *immutability*, and *universality*.
2) He strives for a method that brings only the *truth* and everything from this *truth*.
3) Descartes understands *clarity* and *distinctness* as unambiguous indicators of *truth*, whereby he understands *Mathematical knowledge* as only such *irrefutably clear knowledge*.
4) Intuition and deduction are the two epistemic methods that Descartes understands to be the methodological realizers of finding immutable truths. The knowledge of intuition is constructed by Descartes as the most certain of all knowledge and further knowledge can be deduced from it. Truth thus appears for Descartes as a system, in which individual things and ideas can be put together and related to one another.
5) The *contradiction* appears as a certain indicator of *falsity*, which recalls the work of Parmenides, Plato, and Aristotle and the logic of the binary.

I now draw from the work of Immanuel Kant as a second voice of Enlightenment. His work can be read to a certain extent as a reaction to the dispute between rationalism and empiricism that flared up within modern philosophy (cf. Ruml 2022, 309). In light of this dispute, Kant strives to determine the limits and possibilities of *human cognition* once and for all. This project is implemented most clearly in his main work *Critique of Pure Reason*, to which my reading in this chapter also refers[14].

In his *Critique of Pure Reason*, Kant establishes a strict distinction between different modes of cognition and poses the question of the possibility of certain modes of cognition and disciplines (cf. Hintikka 1967, 364). Two of Kant's distinctions are particularly fundamental. First, he distinguishes a priori knowledge as knowledge independent of experience from *a posteriori*

14 Before the *Critique*, Kant had already developed a theory of mathematics in his *Prize Essay*, which, appears further developed than in the Critique and is why I am working with the Critique here (cf. e.g. Shabel 2021, 2).

knowledge as knowledge from experience (cf. Slavkow 1976, 191)[15]. Second, he differentiates between *analytical* and *synthetic* judgments. Analytic judgments are those that are fed by the object in question itself, for example by naming partial aspects of the definition of the object[16] (cf. Kitcher 1975, 26). Synthetic judgments, on the other hand, make use of the perception of the object in question and deduce a judgment from it; synthetic judgments cannot be explicitly inferred from the object alone (cf. Shabel 2021, 3, 7).

Kant classifies *Mathematical knowledge* as judgments that are synthetic and a priori. Along with his discussion of *Mathematical truth,* he therefore also discusses the possibility of synthetic a priori judgments in general (cf. Felgner 2020, 176).

In the context of differentiating distinctions between *synthetic* and *analytical* and *a priori* and *a posteriori*, Kant elaborates the concept of *Anschauung*:

> Sensory perception is either pure perception (space and time) or empirical perception of that which is immediately presented as real in space and time through sensation. By determining the former, we can obtain a priori knowledge of objects (in mathematics) [...].
> *Critique of Pure Reason, 131 [my translation from German to English]*

Kant describes two types of intuition. Firstly, an intuition in experience, whose judgments are thus a posteriori. And secondly, a *pure intuition* that takes place independently of experience. He understands the objects of *Mathematics* as those that can be understood through this *pure intuition.*

Let us take a closer look at Kant's conception of this *Mathematical* mode of cognition and its attributes:

> [...] mathematical propositions are always judgments a priori and not empirical, because they involve necessity [...]. [...] I restrict my proposition to pure mathematics, whose concept already implies that it does not contain empirical, but merely pure knowledge a priori.
> At first one might think that the proposition $7 + 5 = 12$ is a merely analytical proposition, which follows from the concept of a sum of seven and five

15 A philosophical-historical classification and extensive explanation of the apriori-aposteriori distinction can be found in Russell 2020.

16 A prominent example of an analytical judgment is "Bachelors are unmarried." This judgment refers precisely to the definition of "bachelors" and is therefore analytical. For more examples and explanations, see Russell 2020.

> according to the law of contradiction. However, [...] [t]he arithmetical proposition is [...] synthetic [...].
> Nor is any principle of pure geometry analytic. That the straight line between two points is the shortest is a synthetic proposition. [...]
> *Critique of Pure Reason, 38–40 [my translation from German to English]*

Here Kant first clarifies that his investigations into *Mathematics* refer to *pure Mathematics*, i.e. those whose knowledge arises a priori (independent of experience).

In this context, he also establishes a form of epistemic hierarchy between a posteriori and a priori knowledge by attributing a *necessity* to a priori knowledge that knowledge from experience cannot have. He classifies *Mathematical Knowledge* as 'necessary, a priori knowledge.'

Kant then uses the example of the equation $7 + 5 = 12$ to explain the synthetic character of *mathematical knowledge*. At first, he writes, the equation can be taken to be analytical and one might think that the concept of the number 12 contains the fact that it can be composed of 7 and 5. On closer inspection, however, he argues, it becomes clear that this is not actually the case, but rather that one must imagine the numbers in order to establish knowledge of the equation – one must make use of *intuition*. This means that all arithmetical judgments are synthetic judgments for Kant.

Kant also understands geometric judgments as synthetic. Although there are individual analytical principles in geometry, they only take on a mathematical character because they are "representable in contemplation." Kant thus makes representability in (pure) intuition a necessary criterion of mathematical concepts, judgments, and objects.

The character of synthetic knowledge refers to the idea of *true knowledge* as a system whose connections to further knowledge must be developed.

He goes on to write about the reliability of *mathematical knowledge* and its significance:

> [...] the mathematical, is in the old possession of reliability [...]. Moreover, when one is beyond the circle of experience, one is sure not to be refuted by experience. The attraction of extending one's knowledge is so great that one can only be stopped in one's progress by a clear contradiction which one encounters. This, however, can be avoided if one's deductions are made cautiously, without them remaining less deductions for that reason. Mathematics gives us a splendid example of how far we can go in a priori knowledge,

> independently of experience.
> *Critique of Pure Reason, 33–34 [my translation from German to English]*

First of all, Kant once again emphasizes the special degree of reliability *of mathematical knowledge*. This degree of reliability functions as an ideal for other disciplines; even if they make use of fundamentally different ways of knowing, they should strive for the degree of *epistemic certainty* that Kant ascribes to *Mathematics*.

This affirmation becomes particularly clear in the last sentence of the quoted excerpt: Kant establishes *Mathematics* here as the sphere in which the *epistemic certainty* to which he aspires is fulfilled. This gives it the character of a concept that shapes the epistemic practices of other disciplines and contexts.

The second aspect worth emphasizing here lies in the role that Kant assigns to the idea of *contradiction*: it appears as *the* indicator of the *falsity* of an epistemic practice.

Kant also explicitly discusses this degree of epistemic reliability with regard to *Mathematical axioms*:

> The mathematical axioms (e.g. between two points there can only be a straight line) are even general a priori cognitions and are therefore rightly called principles […]. But I cannot therefore say that I recognize this property of straight lines in general and in itself from principles, but only in pure intuition.
> *Critique of Pure Reason, 280 [my translation from German to English]*

Kant describes *Mathematical axioms* here as findings of a particular general validity, which, however, must also be representable in *pure perception*. Further he writes:

> But to determine a view a priori in space (shape), to divide time (duration), or merely to recognize the generality of the synthesis of one and the same in time and space, and the magnitude of a view in general (number) arising from it, is a matter of reason through the construction of concepts and is called mathematical.
> *Critique of Pure Reason, 568 [my translation from German to English]*

Here Kant conceptually links *Mathematical knowledge* and the idea of pure perception with his concept of *construction,* which is also of central importance for his understanding of *Mathematics*: Kant defines *Mathematics* here as the construction of concepts within 'pure perception.'

Kant expands on this understanding of *Mathematics* and uses it to differentiate between *Mathematics* and *Philosophy*. Kant describes philosophical knowledge here as that which emerges from concepts, while mathematical knowledge consists in the construction of concepts (Critique of Pure Reason, 560–561).

Following this distinction, Kant clarifies his distinction between *empirical* and *pure intuition*: the construction of concepts, which *Mathematics* undertakes, is an activity in *pure intuition*. This pure *intuition is* such because it has a degree of *general validity that empirical intuition* does not have: Kant understands pure Anschauung as an abstract form of Anschauung that contains only the necessary properties of a particular concept (Critique of Pure Reason, 560–561). He understands this abstraction ("the general in the particular") as the characteristic of *Mathematics* and its distinguishing feature from philosophy. What both disciplines have in common, however, is their character as a priori knowledge. He refers to this time and time again in the further course of the *Critique*, as in this section:

> All knowledge of reason is now either from concepts or from the construction of concepts; the former is called philosophical, the latter mathematical.
> *Critique of Pure Reason, 651 [my translation from German to English]*

For Kant, all knowledge of *reason* is therefore either philosophical or mathematical, whereby *Mathematics* constructs concepts and philosophy draws knowledge from them.

In the context of the construction of concepts, pure perception plays a central role, which Kant repeatedly explains in the course of the *Critique:*

> Otherwise, we distinguish by appearances that which is essentially attached to the perception of the same, and is valid for every human sense in general, from that which is only accidental to it, in that it is not valid for the relation of sensuality in general [...]. And here the former cognition is called that which represents the object in itself, but the latter only the appearance of the same.
> *Critique of Pure Reason, 71–72 [my translation from German to English]*

Here Kant emphasizes that *pure intuition* is the intuition that is accessible "to every human sense." Kant thus creates the concept of the "human sense" along the lines of the idea of pure *intuition* and *Mathematics* and defines the "human sense" as the availability of specific knowledge by showing that if a priori knowledge is the knowledge that 'all humans share,' then the disposition to access this knowledge is a necessary characteristic of *being human*. If *Mathematical Knowledge* is the knowledge that *all humans* can access at any time, then the availability of this knowledge is a necessary characteristic of *being human*.

The link between *Mathematical knowledge* and the concept of *the human* becomes increasingly clear in the course of the first *Critique* and is associated with a link between *Mathematics* and *Reason:*

> Mathematics gives the most splendid example of pure reason happily extending itself without the aid of experience. *[...]* Hence pure reason hopes to be able to extend itself as happily and thoroughly in transcendental use as it has succeeded in doing in mathematical use, if it employs there chiefly the same method that has been of such apparent use here.
> *Critique of Pure Reason, 560 [my translation from German to English]*

What has already been hinted at in other quoted passages becomes explicit as Kant establishes *mathematical knowledge* in its a priori certainty as an ideal and model for the use of *pure reason*.

By citing *Mathematics* as a prime example of the use of pure *reason,* he renders *Mathematics* a concept that fundamentally shapes the notion of *reason*. If *Mathematics* is the example of the successful use of pure *reason,* then the idea of *pure reason* is directed towards *Mathematics* through Kant's attempt to establish epistemic certainty as the fulfillment of pure reason as which *Mathematics* appears.

Kant's link becomes even clearer in the following section:

> The very dignity of mathematics (this pride of human reason) rests on the fact that, since it gives reason the guidance to comprehend nature [...] far beyond all expectations of philosophy based on common experience, thereby itself gives occasion and encouragement to the use of reason extended beyond all experience [...].
> *Critique of Pure Reason, 380 [my translation from German to English]*

Here Kant emphasizes that *Mathematics* is a particularly sublime way of knowing and thus also intensifies its conceptual connection to *human reason* and intensifies the *Mathematics-Human* continuum present in his work. Understanding *Mathematics* as the "pride of human reason" means making *Mathematics* a defining concept for both *reason* as an image of thinking and for *the human*[17] as an image of being.

Accordingly, Kant's image of *(Mathematical) thinking* can be summarized through the following core claims[18]:

1) *Mathematics* and the axiomatic method are defined by Kant by their *necessary validity.* The knowledge they produce is construed as fully *timeless* and *reliable knowledge*. Thus, Kant conflates Mathematics with epistemic necessity much like Plato, Aristotle, Descartes, and Ellenberg do.
2) For Kant, *Mathematics* is an ideal of *universal knowledge*. *Mathematics* is constructed by Kant as the sphere in which knowledge of *absolute epistemic reliability* exists to which *pure reason* per se and other disciplines (such as those of metaphysics) should aspire. Therefore, Kant's philosophy bears its very own *Mathematics-Thinking* continuum, making *Mathematics* a refence point for *thinking* more generally.
3) *Mathematical knowledge* is a priori knowledge and therefore independent of the physical world and individual experience. A priori knowledge is the knowledge that *humans* have at their disposal precisely because they are *human*.
4) *Mathematical knowledge* is synthetic knowledge because it makes use of *pure intuition* to derive knowledge. In this dimension, Kant's conception of *Mathematics* is based on the idea of *secure knowledge as a system* and is similar to the philosophies of Parmenides or Plato in this regard.

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17 For a historical and power-critical positioning of the idea of *mathematics* as the "greatest achievement of human rationality", see Hottinger 2016, 13.

18 The respective points can be found as examples in the following secondary literature:
Re 1) Felgner 2020, 183/Kröber 1976, 183/ Shabel 2021, 2, 5, 9/ Slavkow 1976, 192, 199/ Wolff-Metternich 1995, 31, 118
Re 2) Kitcher 1975, 36/Shabel 2020, 5/ Slavkow 1976, 200
Re 3) Hintikka 1967, 352/Kitcher 1975, 26/Slavkow 1976, 191/ Shabel 2021, 4 and Hintikka 1967, 374/ Kitcher 1975, 24, 25/Kröber 1976, 183/Slavkow 1976, 192/Wolff-Metternich 1995, 31
Re 4) Hintikka 1967, 364/ Kitcher 1975, 36 /Peters 1964, 158/Slavkow 1976, 192, 199/Shabel 2021, 2, 7 and Felgner 2020, 177/Slavkow 1976, 192/Wolff-Metternich 1995, 31
Re 5) Felgner 2020, 183/Hintikka 1967, 374/ Kitcher 1975, 24, 25/ Kröber 1976, 183/Slavkov 1976, 192
Re 6) Shabel 2020, 7/Slavkov 1976, 195/Wolff-Metternich 1995, 18

5) For Kant, *mathematical knowledge* appears as *human knowledge* because *Mathematical concepts* are those concepts that are available to *all humans*. In this sense, mathematical knowledge becomes a defining characteristic of *reason* and of *being human* showcasing what I have called the *Mathematics-Rationality-Human* continuum.
6) Kant understands *contradiction* as an objective indicator of *falsity*.

Despite their very different theories and affiliations with fundamentally different schools of thought, Descartes and Kant do share some decisive qualities in their respective understanding of *Mathematics*. These shared aspects can be summarized through the following claims:

1) The claim of a *universal, necessary truth* is established and is a defining epistemic value for *Mathematics*.
2) The universalization of 'human knowledge' occurs through the notion of *Mathematics*. Both believe that there is knowledge that is accessible to *all human beings* because they are *human,* and that *Mathematical knowledge* constitutes such particularly *human knowledge*. Thus, they both actively partake in the construction of the *Mathematics-Human* continuum.
3) The *contradiction* continues to be treated as a reliable indicator of *false* or *unreliable knowledge.*
4) *Mathematics* appears, especially in Kant, but also in Descartes, as the sphere which produces knowledge as the goal of various ways of *using reason*. Mathematics is conceptualized as proof that *the human mind* can produce such knowledge with particular quality and authority.
5) *Reason* and *truth* are further explained in their conceptual context as the former is the path to the latter.
6) Kant and Descartes deepen the idea of *truth* as a system that is closed and in which individual bodies of knowledge are linked to one another.

Core aspects of the *image of (Mathematical) thinking* that we have traced in Parmenides, Plato, and Aristotle as well as in Pinker, Ellenberg, and Brooks are found in Descartes and Kant too. Above all, the notion of *necessary, universal knowledge* remains and finds its own formulation in each theory. and *Mathematics* is constructed as an exemplary figure for the production of *necessary, reliable knowledge. Mathematics* is thus not only one discipline or method, but rather it is an epistemic practice as well as a symbol of the possibility of fulfilling a concept of *truth* that is rigorously based on gaining epistemic authority.

1.5 A Transhistorical Existence
The Thinking-Being of the *Mathematics-Rationality-Human* Continuum

The contemporary and historical studies of this chapter establish a transhistorical existence of *Mathematics* as an Image of Thinking that relies on a distinctly Western notion of proving, which that notions of *Rationality* and *the Human*.

All three stages of this study – the contemporary scholarship, the Ancient Greek philosophies, and the philosophies of the Enlightenment – have established that *Mathematics* as an Image of Thinking is inextricably interwoven with *the Rational* and *the Human*. *Mathematics* shapes what it is *to think rationally* and this capacity to think, according to the Image of Thinking called *Rationality,* is used to construe what it is *to be human*. I understand this interwovenness of *Mathematics* with *Rationality* and *the Human* as the pivotal transhistorical characteristic of *Mathematics*.

The inextricable nature of the interwovenness of *Mathematics, Rationality*, and *the Human* is crucial and results in two findings that are decisive for the continuation of this thesis:

1) *Mathematics, Rationality*, and *the Human* are not separate notions because they only exist in relation to another. Thus, they build a transhistorical continuum, which I refer to as the *Mathematics-Rationality-Human* Continuum. This thesis therefore studies this continuum with special attention to the movements of *Mathematics* in the *Mathematics-Rationality-Human* Continuum.
2) The existence of the *Mathematics-Rationality-Human* Continuum shows that Images of Thinking do not move merely on an epistemic level because through said continuum they are always already related to the mode of being titled *the Human*. So, the *Mathematics-Rationality-Human* Continuum shows that *Thinking* and *Being* too, constitute a continuum rather than separate entities. To understand this *Thinking-Being* Continuum is to understand that an Image of Thinking is always an Image of Being too. To account for the *Thinking-Being* Continuum created by the *Mathematics-Rationality-Human* Continuum I will use the term *Image of Thinking-Being* in continuing this thesis. This variation emphasizes the inextricable interwovenness of *thinking* and *being* as notions and as practices. Furthermore, this variation continues to align with the work of Deleuze. who deliberately did not separate epistemic and ontological questions from one another

(e.g. Deleuze 1968, 169 f and 217 f). This also means that all three players of the continuum discussed above – *Mathematics, Rationality,* and t*he Human* – are images of thinking-being.

Finally, the thinking-being shaped through the *Mathematics-Rationality-Human* Continuum relies on Western ideals and notions, creating a form of silent sovereignty of Western interpretations of thinking-being. This Western sovereignty is implied in the *Mathematics-Rationality-Human* Continuum and shall now be explored in chapter two of this thesis.