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Accident Liability with Rank Dependence[1]

1. Introduction

One of the successes of law-and-economics has been to demonstrate how various liability rules may contribute to the efficient use of resources. However, these results are obtained assuming that decision makers know the probability of accidents. Moreover, it is commonly assumed that objective probabilities of accidents are taken at face value and not transformed to subjective probabilities. Both assumptions are challenged by a growing amount of experimental data. These experiments have in economics and psychology inspired a substantial theoretical effort that so far has had only a minor impact in law-and-economics.

   In this paper I will in section 2 recapitulate some of the results that suggest a revision of our accident (tort) models. In section 3 it is shown that Yaari's dual theory of choice (or the rank dependent expected value model (RDEV)), the expected utility theory (EU) and the expected value theory (EV) all are special cases of the rank dependent expected utility theory (RDEU). In section 4 I study to which extent the efficiency results of the rules of strict liability and negligence obtained in the expected value model (the textbook model) carries over to the rank dependent expected value model. In general, negligence stands out as the better rule.

2. Reasons for rank dependence

There are several reasons for suggesting that the rank dependent expected utility model is appropriate to represent people's choices when facing the possibility of accidents, breaches of contract, malpractice, etc.

   First, intuition and empirical evidence suggest that people have rather vague knowledge about the risk of accidents. Such vagueness may be characterized by the concept of *ambiguity*, a concept that has been of interest to economists ever since Frank Knight (1921) made the distinction between risk and uncertainty. Ambiguity is an intermediate state between ignorance, in which no probability distributions can be ruled out, and risk, in which all but one distribution is ruled out. In a situation of ignorance the decision maker has no information at all concerning the likelihood of potential outcomes. In a situation of risk the decision maker has objective or subjective probabilities of gi-

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ven outcomes. Ambiguity obtains when the decision maker has limited or vague information and knowledge of how outcomes are produced,[2] which often is the case for accidents.

Ellsberg (1961) provided an early demonstration of the importance of ambiguity in decision making, and he showed that uncertainty is not entirely captured by the concept of probability. Empirical evidence indicates that people distinguish between risk and ambiguity (Camerer 1999). Furthermore, studies show that ambiguity aversion and risk aversion are not (highly) correlated, a correlation one would expect if they were just different designations of the same phenomena (Cohen, Jaffray, and Said 1985, and Schoemaker 1982). Both ambiguity avoidance and ambiguity-seeking behaviour have been found in laboratory experiments.[3] Aversion to ambiguity for low probability-of-loss events is indicated in a laboratory experiment by Einhorn and Hogart (1986).

Second, people probably have *preferences about risk*. Such preferences are at the heart of much of Allais' work. At variance with the expected utility theory, people may give more (or less) weight to objective probabilities than what results from the von Neuman-Morgenstern axioms. In discussing the complexity of individual risk psychology Allais (1988, p. 242) states that »[t]he only result which may be considered as totally certain and invariant, is … the strong preference for security in the neighbourhood of certainty when very large sums are at stake.« The risk of a serious accident fits nicely into such a description.

Surveys often underscore Allais' statement, showing that people are more concerned with low-risk/high-loss accidents than what expected utility would suggest. Lichtenstein et al. (1978) demonstrates that people tend to over-assess the fatality risk of low probability events and under-assess that of high-probability events. Significant differences between scientific assessments and public perceptions of environmental risks are also found by the Relative Risk Reduction Strategies Committee (1990). Similar results are presented in Slovic, Fishhoff and Lichtenstein (1992). In an assessment of several surveys of people's attitude toward various types of accidents, the Bri-

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2 Ellsberg (1961, p. 657) defined ambiguity as »a quality depending on the amount, type, reliability and »unanimity« of information, giving rise to one's degree of »confidence« in an estimate of relative »likelihoods« of future events.« See i.a. Becker and Brownson (1964), and Einhorn and Hogarth (1986). Surveying several experimental studies Edwards (1992, p. 5) makes the following comment: »Currim and Sarin […] compared experimental subjects' assessed expected utility models with their prospect theory, weighted utility, and lottery dependent utility models; and Daniels and Keller (1990) […] assessed expected utility and lottery dependent models. Overall, expected utility did about as well as generalised utility models in predicting choices on a hold-out sample of paired comparison choices, even when the problems were structured to induce expected utility property violations. However, the potential for improved predictive performance by generalised expected utility models may still be achieved. For example, Daniels and Keller (1992) […] have explored a choice-based assessment mechanism in which lottery dependent expected utility appears to perform better than expected utility. Also, Shafir et al. (1992) […] proposed an advantage model of choice that outperformed two special cases of expected utility«.

3 See Becker and Brownson (1964) and Einhorn and Hogarth (1986).

tish Health and Safety Executive (1989, p. 14) concludes: »The expectation value [average predicted number of accidents per year] seems almost irrelevant to public concern, which seems to focus on the possibility (however small) of large numbers of casualties.« Furthermore, Viscusi (1992) has shown in his prospective reference theory that overweighting of low probabilities is a natural consequence of a Bayesian learning mechanism of risk assessment.[4] Finally, Camerer and Loewenstein (2004, p. 20) states that »… EU [expected utility] works reasonably well except when probabilities are very low or high. But low-probability events are important in the economy …«

Third, a separate reason for using a model that explicitly treats various attitudes towards risk is that the former gives a clearer representation of how risk per se is evaluated. The concept of risk aversion in the expected utility theory describes a person's dislike for bad outcomes that might or might not obtain, it does not describe how agents eventually transform probabilities.

Although some of the results obtained in the standard law-and-economics tort model are carried over to the rank dependent expected value model, there are important differences.

3. *Definition of preference functions*

The theory of choice of Yaari (1987) is dual to the expected utility theory. Whereas in the expected utility theory the utility of each outcome of a prospect is weighted by the probability of that outcome, in the dual theory the weights of each outcome depend on the rank of that outcome in relation to other outcomes. In Yaari's theory, attitudes towards risk are characterized by a distortion or transformation applied to probability distribution functions, whereas in the expected utility theory attitudes towards risk are characterized by the utility function of wealth. Both theories are special cases of the RDEU theory.

Consider a prospect $(a_1, a_2, …, a_n; p_1, p_2, …, p_n)$, where the outcomes a_j (j=1, 2, …, n) are ordered from the worst to the best, and where the corresponding probabilities are p_j (j=1, 2, …, n). (The outcomes in the tort model discussed below are the accident outcome and the non accident outcome.) In the RDEU theory the cumulative probability p is then transformed according to a function $q(p)$, where $q(0) = 0$ and $q(1) = 1$.[5] An example is the invented s-shaped function in Fig. 1. The diagonal represents the special case where the cumulative probability is not transformed, i.e. the EU case. Each outcome a_j is weighted by

$$h_j(p_1, p_2, …, p_j) = q\left(\sum_{k=1}^{j} p_k\right) - q\left(\sum_{k=1}^{j-1} p_k\right) = q(F(a_j)) - q(F(a_{j-1})). \tag{1}$$

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4 Hornstein (1992) offers, however, an alternative explanation of observed differences in public and expert risk assessment.

5 I here apply Quiggin's (1993) formulation and interpretation of the RDEU model. The existence of ambiguity leads to the same formal model. The RDEU model is an analytically simple version of the more general cumulated prospect theory, see Tversy and Kahneman (1992).

The arguments of the weighting function are values of the cumulative density function of the probability p. In the first term on the right hand side the argument of the probability transformation function is the probability of obtaining an outcome at least as good as a_j. In the second term the argument is the probability of obtaining an outcome at least as good as a_{j-1}. A numerical example is given in Table 1 and illustrated in Fig. 1.
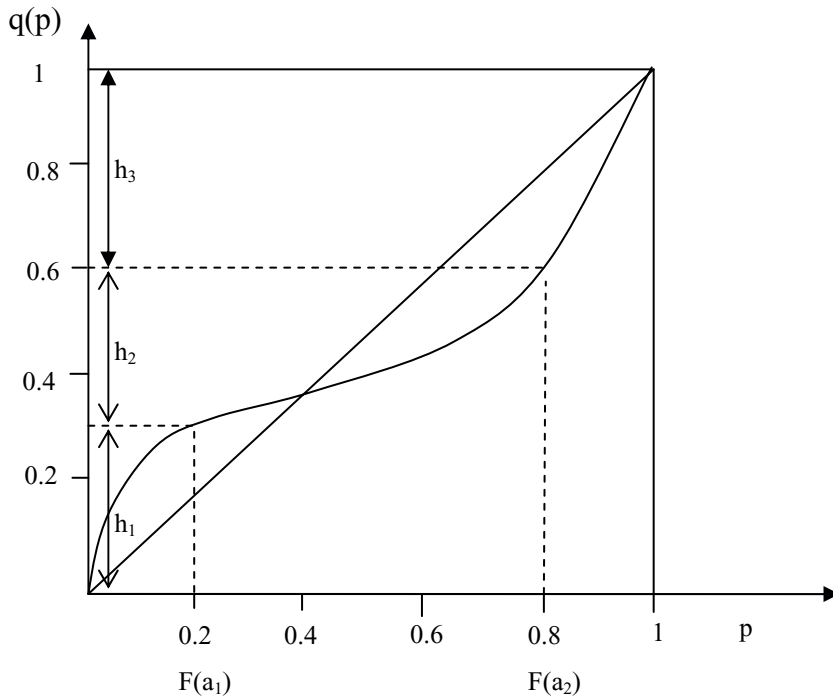

Table 1 Example of probability weighting in the RDEU theory

Outcomes, a_j	a_1	a_2	a_3
Probabilities, p_j	0.2	0.6	0.2
Cumulative probabilities, $F(a_j)$	0.2	0.8	1
Probability weights, h_j	0.3	0.3	0.4

The dual theory and the expected utility theory are special cases of the more general rank dependent expected utility theory where the decision maker maximises

$$U_{RDEU} = \sum_{j=1}^{n} h_j(p_1, p_2, \ldots, p_j) U(a_j).$$

If $h_j(.) = p_j$, (j = 1,2,…,n), the rank dependent expected utility is reduced to expected utility,

$$U_{EU} = \sum_{j=1}^{n} p_j U_j(a_j).$$

Yaari's dual theory of choice, which I here call the theory of *rank dependent expected value*, is based on the special case of the RDEU model where monetary outcomes are substituted for utilities, i.a.

$$U_{RDEV} = \sum_{j=1}^{n} h_j(p_1,p_2,…,p_j)a_j, \text{ or } \sum_{j=1}^{n} [q(F(a_j)) - q(F(a_{j-1}))]a_j.$$

Substituting further probabilities for weighting functions results in the expected value

$$U_{EV} = \sum_{j=1}^{n} p_j a_j, \text{ which is the preference function commonly used in tort models.}$$

4. *Precaution for various preference functions*

The effect of the rules of strict liability and negligence on the level of care will be discussed in the models of expected value (*EV*) and the rank dependent expected value (*RDEV*).[6]

4.1 Expected value

4.1.1 Social costs

I take as a starting point the simple textbook model of unilateral care where x is the injurer's precaution costs, D is the (constant) damage to the victim, p is the probability of an accident, and $p(.)$ is a twice differenciable falling convex function. The expected social cost related to an accident is defined as $x + p(x) D$. The precaution costs that minimizes these costs, denoted x^*, is obtained from the first order condition[7]

$$1 = -p'(x^*)D, \text{ or} \tag{2a}$$

$$p'(x^*) = -\frac{1}{D}. \tag{2b}$$

Equation (2a) is interpreted as follows:

Precaution should increase up to the level where last EURO used on precaution produces a decrease in expected damage equal to one EURO.

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6 The paper presented at the workshop included analyses of the models of expected utility and rank dependent expected utility. Although the rank dependent expected value model is a only special case of the rank dependent expected utility model, it illustrates some of the main characteristics of the latter.

7 An interior solution obtains if $p'(0) < -\frac{1}{D}$, and a corner solution obtains if $p'(0) \geq -\frac{1}{D}$. In the rest of the paper only interior solutions will be discussed.

4.1.2 Injurer's choice

Strict liability
Under strict liability the injurer is liable for all social costs (accident costs are internalized). Let W be the exogenous wealth of the injurer. The expected value of the injurer's wealth is

$EU_{EV} = p(x)(W - x - D) + (1 - p(x))(W - x) = W - [x + p(x)D]$.

Assuming that the injurer is maximizing the expected value, that he knows $p(x)$ and D with certainty, and that there are no litigation costs, differentiation of the expected value w.r.t. x gives the first order condition

$1 = -p'(\tilde{x})D$, or   (3a)

$p'(\tilde{x}) = -\dfrac{1}{D}$ (3b)

where $\tilde{x}$ denotes the optimal precaution costs.[8] From (2a) and (3a) it is seen that $p(\tilde{x})$ = $p(x^*)$. Thus, under strict liability, the injurer will choose $\tilde{x} = x^*$, i.e. the socially optimal precaution.

Equation (3a) may be interpreted as follows:

Under strict liability an injurer will increase precaution costs up to the level where last EURO used on precaution produces a decrease in expected damage equal to one EURO.

Negligence
Under negligence, denote due care $\bar{x}$, let this value be known with certainty by the injurer, and assume $\bar{x} = x^*$. If the injurer chooses a level of precaution equal to due care, his costs will consist only of precaution costs, whereas for a lower level of care also damage costs will be included. Therefore, his costs will be minimized by satisfying due care, which is socially optimal.

4.2 Rank dependent expected value

The model of rank dependent expected value may represent two different situations. Either the injurer is weighting the probability in the manner Allais has suggested, or if ambiguity prevails, the injurer acts *as if* he is weighting probabilities.

4.2.1 Social costs

In order to determine social costs one has to decide whether also potential victims are weighting the probability of an accident, and eventually to which degree. Two main alternatives might be considered. One alternative is that potential victims in fact do not consider the possibility of the accident. One might for instance argue that a costumer, at variance with the shop-keeper, does not consider the possibility of being hurt by a badly attached lamp in a shop. The possibility of a lamp falling down on a costumer is so remote that the costumer does not even consider the possibility. In this case it seems

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8 The second order condition $\dfrac{d^2(EU_{EV})}{dx^2} = -p''(x)D < 0$ is clearly satisfied.

unreasonable to assume that potential victims are weighting probabilities. Social costs should then be defined as in section 4.1.1.

Another alternative is that potential victims *do* consider the possibility of the accident, and that they will be weighting the probability of the accident, either because of ambiguity or because they have a dislike for low probability/high cost events.

As a special case, let there be only one potential victim that is weighting the probability of an accident according to a function $r(p(x))$. The social cost to be minimized is $x + r(p(x))D$, and the social optimal precaution x_{RD}^* (subscript RD for rank dependence) is determined by the first order condition

$$r'(p(x_{RD}^*))p'(x_{RD}^*) = -\frac{1}{D}. \tag{4}$$

If the probability of an accident is being overweighted, i.e. $r' > 1$, the optimal value x_{RD}^* will be higher than x^*, the probability obtained above when probabilites were not weighted.

The special case with just one potential victim is rather unlikely in practice. There might be several potential victims with various degrees of knowledge. Their weighting functions might differ, and the expected social costs difficult to determine, both in theory and in practice.

Even if potential victims are weighting probabilities of accidents, there is an argument for assuming that x^* should be considered as the socially optimal level of precaution. In welfare theory it is sometimes argued that the social planner should make corrections for individual preferences that are based on wrong beliefs. Taxes on some »bads« (cigarettes, alcohol,…) is an example. To the extent that the victims are weighting probabilities because of lack of knowledge one may consider such weighting as irrelevant for social costs. However, if the weighting is not a result of lack of knowledge, but rather a result of clear preferences concerning low probability/high loss events (cf. Allais) this argument is less relevant.

In the following, both definitions of social costs will be considered.

4.2.2 Injurer's choice

In our benchmark model (accident or not) the weighting functions (1) take the simple forms

$h_1(.) = q(p) - q(0) = q(p)$
$h_2(.) = q(1) - q(p) = 1 - q(p)$.

In this simple case with only two alternatives the weighting of probabilities is *formally* similar to a standard weighting of probabilities (for example transformation from objective to subjective probabilities), where $f_1 = q_1$ and $f_2 = 1 - f_1 = 1 - q_1$. The interpretation, however, is different. In standard weighting of probabilities, the probabilities are not weighted according to the ranking of the outcomes. For instance, low probabilities might be overweighted regardless of how good the associated outcomes are in comparison with other outcomes. The ranking is essential either if injurers react to ambiguity or if they behave according to Allais' assessment that people tend to overweigh low probabilities of the worst outcomes.

Strict liability

If the rule is strict liability, an agent with a probability weighting function $q(p(x))$ will maximize $U_{RDEV} = W - [x + q(p(x))D]$ with respect to x. The first order condition is

$$1 = -q'(p(\tilde{x}))p'(\tilde{x})D \text{ or} \tag{5a}$$

$$q'(p(\tilde{x}))p'(\tilde{x}) = -\frac{1}{D} \tag{5b}$$

where $\tilde{x}$ now is the optimal value of the injurer's precaution costs under strict liability in the case of rank dependent expectation.[9]

The first order condition (5a) can be interpreted as follows:

The injurer will increase precaution up to the level where the last EURO used on precaution produces a decrease in the weighted expected damage of one EURO.
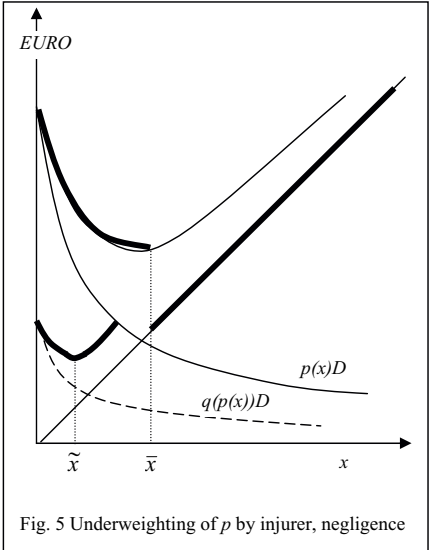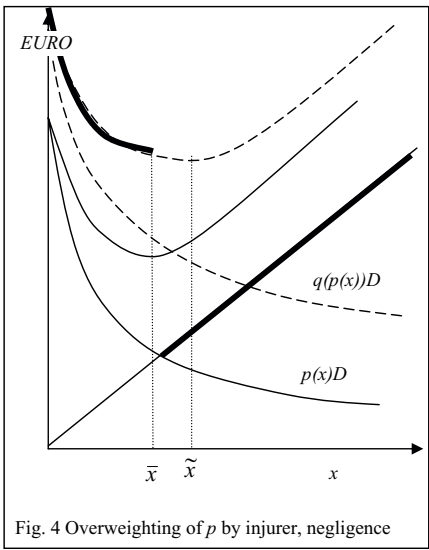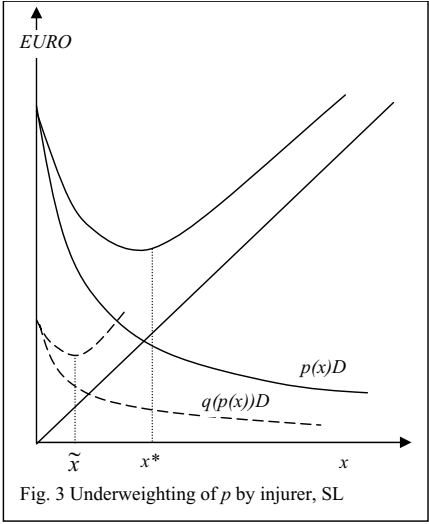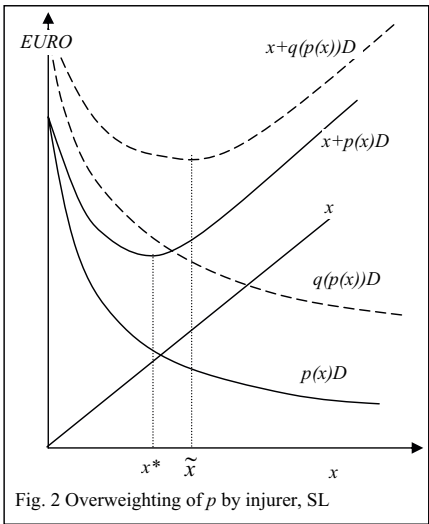
Will the injurer's choice be socially optimal? Obviously, the answer depends on how the socially optimal level of care is determined. Let us first assume that this level is the one determined by (2a), i.e. x^*.

From (2a) and (5a) it is seen that $\tilde{x} = x^*$ if $q' = 1$ at $\tilde{x}$. For this particular value of q', which implies that the probabilities are not weighted, strict liability will lead a cost minimizing agent to minimize social cost.

If $q' > 1$ at $\tilde{x}$, $p'(\tilde{x}) < p'(x^*)$ and $\tilde{x} > x^*$. The injurer will take too much precaution, i.e. too much in relation to what is required to minimize social costs, see Fig. 2, where the lower curves represent social costs and the upper (dotted) curves represent the injurer's weighted costs. If $q' < 1$ at $\tilde{x}$, the injurer will take too little precaution, see Fig. 3. Strict liability is clearly not socially efficient. The empirical finding that people tend to overweight low probabilities of bad outcomes (or have aversion against ambiguity) suggests that strict liability will tend to produce too much precaution.

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9 The second order condition requires that $q''(p(x))[p'(x)]^2 - q'(p(x))p''(x) \geq 0$, or $q''(p(x))$
$\geq -\dfrac{q'(p(x))p''(x)}{[p(x)]^2}$ which means that the weighting function must be convex or not »too concave« at $\tilde{x}$.

Fig. 2 Overweighting of p by injurer, SL

Fig. 3 Underweighting of p by injurer, SL

Fig. 4 Overweighting of p by injurer, negligence

Fig. 5 Underweighting of p by injurer, negligence

Assume now that the socially optimal level of care is x_{RD}^*, as determined by (4), and furthermore that the injurer and the victim are weighting probabilities in the same manner, i.e. $r(p(x)) = q(p(x))$. Then the rank dependent expected social costs are equal

to the injurer's rank dependent expected costs, and the injurer will choose the socially optimal level of precaution, i.e. $\tilde{x} = x_{RD}*$ (like in the expected value model).

If the two parties are weighting probabilities differently, the level of precaution chosen by the injurer might be either lower or higher than what is socially optimal.

Negligence

The effect of the rule of negligence depends on how the judge considers rank dependence. There are four cases to consider.

(i) Assume first that the socially optimal level of care is $x*$ and that due care $\bar{x}$ is set at this level. If the injurer is overweighting the probability of an accident (dashed curves in Fig.4), he will according to his own beliefs consider $\tilde{x}$ to be his cost minimizing precaution. He will, however, in order to be absolved of liability, choose due care, which is socially efficient. If the injurer is underweighting the probability of an accident, he might, or might not, choose this level of care. If underweighting is substantial, so that $\min[x + q(p(x))D] < \bar{x}$, as illustrated in Fig. 5, he will stick to his own preferences (based on weighted probabilities or ambiguity), and choose less precaution than what minimizes social costs, i.e. $\tilde{x} < \bar{x} = x*$. On the other hand, if there is a moderate underweighting such that $\min[x + q(p(x))D] \geq \bar{x}$ the injurer will choose $\bar{x} = x*$.

Negligence is thus efficient if the injurer is overweighting or only moderately underweighting the probability of an accident.

(ii) Assume as above that the socially optimal level of care is $x*$, but that the judges are weighting probabilities in the same manner as injurers. An injurer will then be absolved of liability by choosing $\tilde{x} = x_{RD}*$, which in this case is not socially optimal. As in case (i) the injurer might or might not choose to be absolved, but in neither alternative the socially optimal level of care, $x*$, will be chosen (except by coincidence).

(iii) If the socially optimal level of care is $x_{RD}*$, and that due care is set at this level, the injurer's rank dependent expected costs and the social expected costs are identical, and the result is analogous to the one in the model of expected value. The injurer will choose the socially optimal level of care.

(iv) If the socially optimal level of care is $x_{RD}*$, whereas the level of due care determined by the judge is $x*$, the injurer will in general not choose the optimal level of precaution.

5. Conclusions

Laboratory experiments often show that some people do not behave according to maximization of expected utility. It should therefore be an ambition of the profession to apply non expected utility theories to various fields of economic theory. In this paper the effect of rank dependence has been applied to the accident model of unilateral care.

In the standard expected value model both strict liability and negligence produce socially efficient (cost minimizing) results. In the rank dependent expected value model the effects of the two tort rules depend on how social costs of accidents are defined. If social costs are defined as the sum of precaution costs and expected damage costs (without weighting of probabilities) strict liability will in general not produce socially ef-

ficient precaution. Negligence will produce socially efficient precaution only when (1) the injurer is overweighting or moderately underweighting the probability of an accident, and (2) the judges determining the standard of precaution do not weight probabilities.

If the social costs are determined by weighting of probabilities, strict liability will produce socially efficient solutions only if the weighting function is identical for the victim and the injurer. Under the rule of negligence the injurer will choose the socially optimal level of precaution if the judge is determining due care at the level of precaution where the weighted social costs are minimized.

As a whole, the rule of strict liability, more than the rule of negligence, loses its appeal when ambiguity and weighting of preferences are taken into account.

References

Allais, Maurice (1988): The General Theory of Random Choices in Relation to the Invariant Cardinal Utility Function and the Specific Probability Function. In: Munier, B. (ed.), *Risk , Decision and Rationality*, Dordrecht, Reidel, 233-289.

Becker, S.W., and F.O. Brownson (1964), »What Price Ambiguity? Or the Role of Ambiguity in Decision-making«, *The Journal of Political Economy*, *72*, 62-73.

Camerer, Colin (1999), »Ambiguity Aversion and Non-additive Probability: Experimental Evidence Models, and Applications«, in: Luigi Luini (ed.), *Uncertain Decisions: Bridging Theory and Experiment* (Boston/Dordrecht/London: Kluwer Academic Publishers).

Camerer, Colin F. and George Loewenstein (2004): »Behavioral Economics: Past, Present and Future«. In Colin F. Camerer, George Loewenstein and Matthew Rabin: *Advances in Behavioral Economics*, 3-51. Princeton and Oxford, Princeton University Press.

Cohen, Michèle, Jean-Yves Jaffrey, and T. Said, (1985): »Individual Behavior under Risk and Uncertainty: An Experimental Study«, *Theory and Decision*, *18*, pp. 203-28.

Currim, I. S. and R. Sarin (1992): Robustness of Expected Utility Models in Predicting Individual Choices. *Organizational Behavior and Human Decision Processes,* 52, 544-568.

Daniels, R. and L. R. Keller (1990): An Experimental Evaluation of the Descriptive Validity of Lottery Dependent Utility Theory. *Journal of Risk and Uncertainty*, *3*, 115-134.

Daniels, R. and L. R. Keller (1992): Choice-based Assessment of Utility Functions. *Organizational Behavior and Human Decision Processes*, *52*, 524–543.

Edwards, Ward, ed. (1992): *Utility Theories: Measurement and Applications. Studies in Risk and Uncertainty*. Dordrecht, Kluwer.

Einhorn, H.J., and R.M. Hogart, (1986), »Decision Making under Ambiguity«, *The Journal of Business,* 225-55.

Ellsberg, Daniel, (1961), »Risk, Ambiguity, and the Savage Axioms«, *Quarterly Journal of Economics*, *75*, 643-69.

Hornstein, D.T. (1992): Reclaiming Environmental Law: A Normative Critique of Comparative Risk Analysis. *Columbia Law Review*, *92,* 562–633.

Knight, Frank (1921): *Risk, Uncertainty and Profit*. Boston, Houghton Mifflin.

Lichtenstein, Sarah, et al. (1978) Judged Frequency of Lethal Events. *Journal of Experimental Psychology*, *4*, 551-578.

Quiggin, John (1982): A Theory of Anticipated Utility. *Journal of Economic Behaviour and Organisation*, *3*, 323-343.

Quiggin, John (1993): *Generalized Expected Utility Theory: The Rank Dependent Model*. Academic Publishers, Boston/Dordrecht/London.

Relative Risk Reduction Strategies Committee, U.S. EPA Science Advisory Board (1990): *Reducing Risk: Setting Priorities and Strategies for Environmental Protection.*

Shafir, E.B., D. N. Osherson, and E. E. Smith (1992): An Advantage Model of Choice. *Journal of Behavioral Decision Making*, *2*, 1-23.

Schoemaker, Paul (1982), »The Expected Utility Model: Its Variants, Purposes, Evidence, and Limitations«, *Journal of Economic Literature*, *20*, 529-63.

Slovic P., B. Fishop, and C. Lichtenstein (1982): Facts versus Fears: Understanding Perceived Risks. In: D. Kahneman, P. Slovic, and A. E. Tversky (eds.) *Judgement Uncertainty: Heuristics and Biases*.

Tversky, A. and D. Kahenman (1992): »Advances in Prospect Theory: Cumulative Representation of Uncertainty«, *Journal of Risk and Uncertainty*, *5*, 297-323.

Viscusi W. Kip (1992): *Fatal Tradeoffs: Public and Private Responsibilites for Risk*. New York and Oxford, Oxford University Press.

Yaari, M. E., (1987), »The Dual Theory of Choice Under Risk«, *Econometrica*, *63*, 95-115.